# SEISMIC PERFORMANCE ASSESSMENT USING RESPONSE SURFACE METHODOLOGY

*Florenta-Nicoleta TANASE*[1]
[1] Scientific Researcher, NIRD "URBAN-INCERC", INCERC Bucharest Branch,
ing_tanasenicoleta@yahoo.com

**ABSTRACT**

The Response Surface Methodology is a useful way for assessing the seismic performance of structures. A description of this methodology and a short introduction into the Design of Experiments are presented. By using the cumulative probability density obtained after 10,000 Monte Carlo simulations on the response surface functions, it is illustrated, by a case study, the way in which the probability of a system of being in a damage state can be estimated.

*Keywords*: response surface methodology; seismic performance; probabilistic assessment; cumulative probability density

**REZUMAT**

Metodologia Suprafețelor de Răspuns este o modalitate utilă de evaluare a performanței seismice a structurilor. În articolul de față este prezentată o descriere a acestei metodologii şi o scurtă introducere în Planificarea Experimentelor. Este ilustrat, printr-un studiu de caz, modul în care poate fi estimată probabilitatea ca un sistem să se afle într-o stare de avariere, folosind densitatea cumulată a probabilităților obținută după 10 000 de simulări Monte Carlo asupra funcțiilor suprafețelor de răspuns.

*Cuvinte cheie*: metodologia suprafeței de răspuns; performanță seismică; evaluare probabilistică; densitatea cumulativă de probabilitate

## 1. INTRODUCTION

### *1.1. Response surfaces using metamodels*

Seismic performance of structures at future earthquakes cannot be known with precision, mainly because earthquakes are random phenomena and the structures contain a series of uncertainties.

For a building structure, the most important uncertainties are considered those concerning the materials. By using the "Design of Experiments" approach for the Response Surface Methodology, metamodels are obtained by selecting the parameters having the highest influence on the behaviour of the structural system.

The metamodel is a statistical approximation of complex phenomena, using the characteristics (the input variables) that influence the response of a system. The response is estimated as a function of input variables. The relationship between the response $y$ and the random variables $\xi$ of a system can be expressed by the following equation:

$$y = f(\xi) \tag{1}$$

A metamodel, $g(\xi)$, estimating the relationship $f(\xi)$ between the response and input variables vectors, will become:

$$y = g(\xi) + \varepsilon \tag{2}$$

where $\varepsilon$ is the total error, which is equal to zero when performing computer analyses. The estimated value of response function is:

$$E[y] = g(\xi) \tag{3}$$

The creation of a metamodel is a three-step process: (1) choosing the input variables $\xi$ of the systems which are necessary in estimating the response $y$, (2) choosing the metamodel function $g(\xi)$ and (3) acquiring the data obtained after performing the analyses and fitting the metamodel to them.

### *1.2. Response Surface Methodology*

The Response Surface Methodology (RSM) emerged in the '30s. Box and Wilson (1951) developed this methodology in the field of chemistry research. Nowadays the RSM is applied in many research fields such as: aerospace engineering, structural reliability, chemical and industrial engineering etc.

The RSM implies obtaining response surfaces using a function of n variables and computing the polynomial coefficients (Myers and Montgomery, 2002), the response surface being a polynomial regression. If the number of variables is large a design of experiments that requires a reasonable number of analyses will be used.

## 2. THE DESIGN OF EXPERIMENTS

The Design of Experiments consists in choosing a set of points in which the response needs to be determined. Several types of designs are available, such as: *Full Factorial Design* (*FFD*), *Central Composite Design* (*CCD*), Box-Behnken Design, Taguchi orthogonal matrices etc.. The most used designs remain the FFD and the CCD.

The FFD is used to decide which factors influence a dependent variable. The CCD is an option when experimental investigations are performed for each possible combination of the factors levels.

A factor is an independent variable that can have several levels which are values that can be used for it. A factor must have at least two levels in order to discover its influence.

The number of analyses to be performed can be substantially reduced using the appropriate design.

The use of standardized or coded form of the variables ( $x_i$ ) is much more suitable than their actual values ( $\xi_i$ ).

*Full Factorial Design* or $3^k$ Factorial Design is the simplest design, where the variables are given three coded values: -1, 0 and +1. In the Complete Factorial Design the responses are obtained using all possible combinations of the three values (levels) of $k$ variables. The number of combinations (*design points*) will be $N = 3^k$, which can become too large when many variables are considered.

In order to obtain a sufficient degree of precision for the results while using a smaller number of design points other designs have been created, such as Central Composite Design.

The Central Composite Design is in fact a $2^k$ Full Factorial Design, the levels of the variables being the values -1 and +1, representing points on the Central Composite Design cube. Each variable has a corresponding axis with two points situated at the distance $\alpha$ ($\alpha \geq 1$) from the center of the cube.

The value of $\alpha$ has influence on the rotatability property of CCD, which gives a constant variation of the response estimated at a fixed distance from the central point. The number of central points $n_0 \geq 1$. The CCD is rotatable if $\alpha = \left(2^k\right)^{1/4}$.

If $\alpha > 1$, the variable considered needs to be evaluated at 5 levels: $-\alpha$, -1, 0, +1, $+\alpha$, but this is not possible when the values cannot be outside de lower and upper bonds. In such cases $\alpha = 1$, and the results offer a very good estimate. The number of total points needed becomes $N = 2^k + 2k + 1$.

Figure 1 presents a comparative graphic representation of FFD and CCD, with $\alpha = 1$, while using three variables $X_1$, $X_2$, $X_3$.

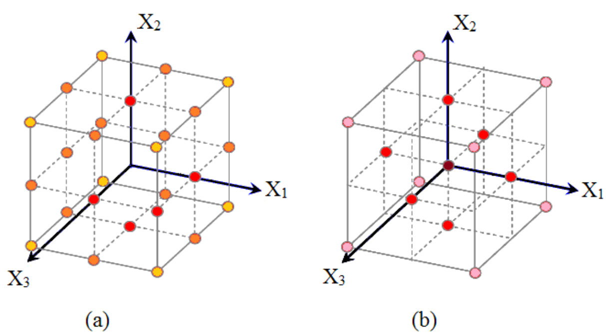


**Fig. 1.** Graphic representation of (a) Full Factorial Design and (b) Central Composite Design for $\alpha = 1$ with three variables

## 3. RESPONSE SURFACE FUNCTIONS

The model used to obtain the response surfaces contains the vector of variables considered and the vector of results from the analyses performed. The response surface functions have polynomial form.

A drawback of RSM is the limited number of variables considered which is limited at eight when using the Design of Experiments. Therefore, only variables with significant influence on the response should be used.

The response surface functions are usually first or second-degree polynomials, because they contain fewer terms and the number of analyses to be performed is reduced.

The response surface function for a second degree polynomial model is:

$$y=\beta_0+\sum_{i=1}^{k}\beta_i x_i+\sum_{i=1}^{k}\beta_{ii} x_i^2+\sum_{i=1}^{k-1}\sum_{j>1}^{k}\beta_{ij} x_i x_j+\varepsilon \quad (4)$$

where

$y$ is the system response;

$x_i$, $x_j$ are the independent variables with their normalized form;

$\beta_0$, $\beta_i$, $\beta_{ii}$, $\beta_{ij}$ are the unknown coefficients;

$\varepsilon$ is the error;

$k$ is the number of considered variables.

Although the equation (4) contains higher-order terms, a linear regression model can be used to replace it:

$$y=\beta_0+\sum_{i=1}^{n-1}\beta_i z_i+\varepsilon \quad (5)$$

where $n$ is the number of parameters to be estimated and $z_i$ represents the variables in the vector that replaces the vector of initial variables $x_i$ that contained quadratic terms. A function of three variables ($x_1$, $x_2$ and $x_3$)

$$\begin{aligned}y&=\beta_0+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3+\beta_{11}x_1^2+\dots\\&+\beta_{22}x_2^2+\beta_{33}x_3^2+\beta_{12}x_1x_2+\beta_{13}x_1x_3+\beta_{23}x_2x_3+\varepsilon\end{aligned} \quad (6)$$

can be transformed into a linear regression model:

$$\begin{aligned}y&=\beta_0+\beta_1 z_1+\beta_2 z_2+\beta_3 z_3+\beta_4 z_4+\dots\\&+\beta_5 z_5+\beta_6 z_6+\beta_7 z_7+\beta_8 z_8+\beta_9 z_9+\varepsilon\end{aligned} \quad (7)$$

The linear model can be expressed in a matrix form as:

$$Y=Z\beta+\varepsilon \quad (8)$$

where:

$Y=\begin{bmatrix}y_1\\y_2\\.\\.\\.\\y_N\end{bmatrix}_{N\times1}$ is the vector of structural responses with expectation $E[Y]=Z\beta$.

$\beta=\begin{bmatrix}\beta_0\\\beta_1\\.\\.\\.\\\beta_{n-1}\end{bmatrix}_{n\times1}$ is the vector of unknown parameters,

$Z=\begin{bmatrix}1 & z_{11} & z_{12} & \cdots & z_{1,n-1}\\1 & z_{21} & z_{22} & \cdots & z_{2,n-1}\\. & . & . & & .\\. & . & . & & .\\. & . & . & & .\\1 & z_{N1} & z_{N2} & \cdots & z_{N,n-1}\end{bmatrix}_{N\times n}$ is the matrix of constants,

$\varepsilon=\begin{bmatrix}\varepsilon_1\\\varepsilon_2\\.\\.\\.\\\varepsilon_N\end{bmatrix}_{N\times1}$ is the errors vector with the expectation $E[\varepsilon]=0$.

The parameters of the polynomial function can be determined through regression using the least squares method and by selecting the values $(b_0, b_1, \dots, b_{n-1})$ for the unknown parameters $(\beta_0, \beta_1, \dots, \beta_{n-1})$, so that the sum of squares of the differences between the actual structural responses ( $y$ ) and those that were estimated is minimum. The least squares method can be applied as shown:

$$S(b) = \sum_{r=1}^{N} \left( y_r - \hat{y}_r(b) \right)^2 \tag{9}$$

where $S$ is the sum of squares function, $N$ is the number of points considered in the experiment ( $N > n$ ) and $b$ is the vector of least squares that estimates $\beta$ .

By solving the following matrix equation, the polynomial parameters can be estimated (Box and Draper, 1986):

$$b = (Z'Z)^{-1}(Z'Y) \tag{10}$$

The fitted response surface function will be:

$$\hat{y} = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j>1}^{k} b_{ij} x_i x_j \tag{11}$$

Including the Monte Carlo simulation while using RSM makes the process of developing cumulative distribution of probability curves a lot less time-consuming, due to the fact that simulations are performed over the polynomial equation and not before running analyses.

## 4. CASE STUDY

A SDOF system (Fig. 2) was chosen in order to illustrate the use of RSM. The variables that describe the properties of the system are the mass ($x_1$) and the stiffness ($x_2$), while PGA ($x_3$) is the control variable. The damping ratio of the system is 5%.

Sets of five accelerograms, scaled at the chosen levels of PGA (Table 1), were used in nonlinear time-history analysis performed on the SDOF systems.

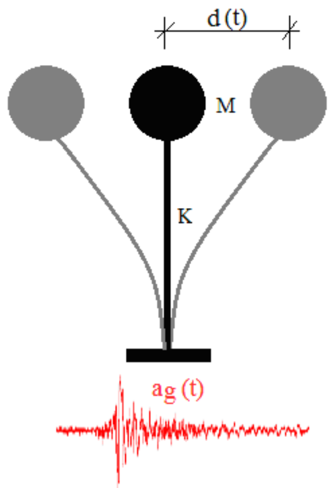


**Fig. 2.** SDOF system

**Table 1.** *Input variables for the Response Surfaces of a SDOF system*

| Random Structural Parameters | Input variables | Lower Bond | Center Points | Upper bond |
|---|---|---|---|---|
| Mass, M | $\xi_1$ (kN . s$^2$/m) | 2100 | 2700 | 3300 |
| | $x_1$ | -1 | 0 | 1 |
| Stiffness, K | $\xi_2$ ( kN/m) | 40000 | 50000 | 60000 |
| | $x_2$ | -1 | 0 | 1 |
| Peak ground acceleration, PGA | $\xi_3$ (g) | 0.12 | 0.30 | 0.48 |
| | $x_3$ | -1 | 0 | 1 |

The recorded response of the system was the top displacement.

The Design of Experiment used CCD resulting 15 combinations of the three variables (Table 2). The minimum value considered for PGA was 0.12g, the medium value vas 0.30g and the maximum 0.48g.

The top displacement was recorded for each one of the time-history analyses and a normal distribution of its value was considered.

Mean and standard deviation of the top displacement of SDOF system can be approximated by a second-degree polynomial expression. The polynomial coefficients can be determined using equation (10).

**Table 2.** *Matrix of experiment and recorded responses*

| Case No. | Parameters | | | Top displacement (cm) | |
|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | Mean, $\hat{d}_\mu$ | Standard deviation, $\hat{d}_\sigma$ |
| 1 | -1 | 0 | 1 | 59.97 | 4.35 |
| 2 | 0 | -1 | 1 | 60.88 | 6.41 |
| 3 | 0 | 1 | 1 | 59.44 | 3.94 |
| 4 | 0 | 1 | -1 | 16.76 | 1.51 |
| 5 | 1 | 1 | 0 | 37.78 | 3.01 |
| 6 | 0 | -1 | -1 | 16.16 | 1.75 |
| 7 | -1 | -1 | 0 | 38.19 | 3.44 |
| 8 | 1 | -1 | 0 | 41.09 | 5.34 |
| 9 | 1 | 0 | 1 | 60.51 | 6.48 |
| 10 | 0 | 0 | 0 | 37.81 | 3.08 |
| 11 | 0 | 0 | 0 | 37.81 | 3.08 |
| 12 | 1 | 0 | -1 | 16.08 | 1.76 |
| 13 | 0 | 0 | 0 | 37.81 | 3.08 |
| 14 | -1 | 1 | 0 | 37.66 | 4.29 |
| 15 | -1 | 0 | -1 | 16.73 | 1.34 |

The response surface models for the mean ($\hat{d}_\mu$) and standard deviation ($\hat{d}_\sigma$) of the response are:

$$\hat{d}_\mu = 37.81 + 0.36x_1 - 0.58x_2 + 21.88x_3 + 0.44x_1^2 - \ldots$$
$$-0.70x_1 \cdot x_2 + 0.43x_2^2 + 0.30x_3 \cdot x_1 - 0.51x_3 \cdot x_2 + 0.07x_3^2 \tag{12}$$

$$\hat{d}_\sigma = 3.08 + 0.40x_1 - 0.52x_2 + 1.85x_3 + 0.51x_1^2 - \ldots$$
$$-0.79x_1 \cdot x_2 + 0.43x_2^2 + 0.43x_3 \cdot x_1 - 0.55x_3 \cdot x_2 - 0.11x_3^2 \tag{13}$$

The response surface models at various levels of seismic intensity were obtained by assessing the polynomial functions for values corresponding to the control variable $x_3$, which represents the normalized form of PGA. Monte Carlo simulations were performed on these models generating random values for the variables $x_1$ and $x_2$, between the lower and the upper bonds, considering their specific distributions of values. The mass and stiffness were considered variables with a uniform distribution of values.

Damage probabilities conditioned by a certain value of PGA, e.g. 0.24g, can be computed after determining the normalized value of $x_3$:

$$x_3 = \frac{0.24g - \frac{0.48g + 0.12g}{2}}{\frac{0.48g - 0.12g}{2}} = 0.5$$

By replacing $x_3$=0.5 corresponding to PGA=0.24g, the response surface models become:

$$\hat{d}_\mu\Big|_{ag=0.24g} = 48.77 + 0.51x_1 - 0.83x_2 + \ldots$$
$$+0.44x_1^2 - 0.70x_1 \cdot x_2 + 0.43x_2^2 \tag{14}$$

$$\hat{d}_\sigma\Big|_{ag=0.24g} = 3.98 + 0.62x_1 - 0.80x_2 + \ldots$$
$$+0.51x_1^2 - 0.79x_1 \cdot x_2 + 0.43x_2^2 \tag{15}$$

Considering a normal distribution of top displacements of SDOF system, 10.000 sample values were generated for the $x_1$ and $x_2$ variables and the estimated responses and their cumulative probability densities were computed (Figures 3 and 4).

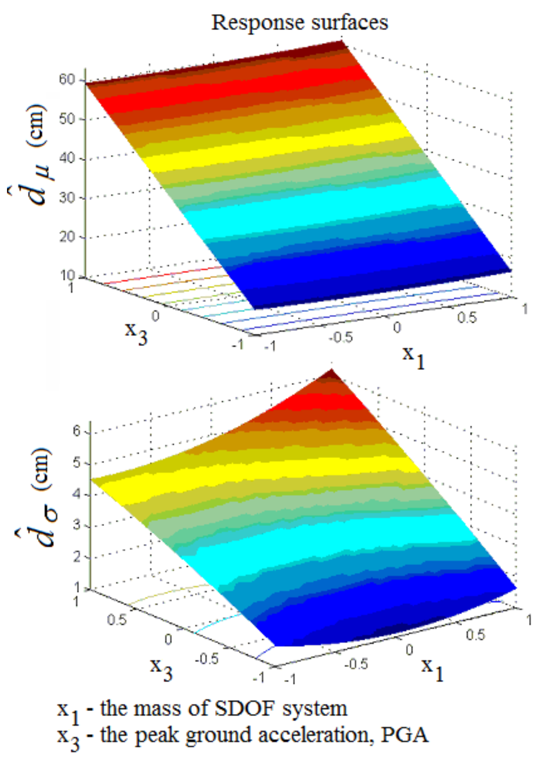


**Fig. 3.** Response surfaces for mean and standard deviation of the top displacement of SDOF system, as a function of mass and PGA

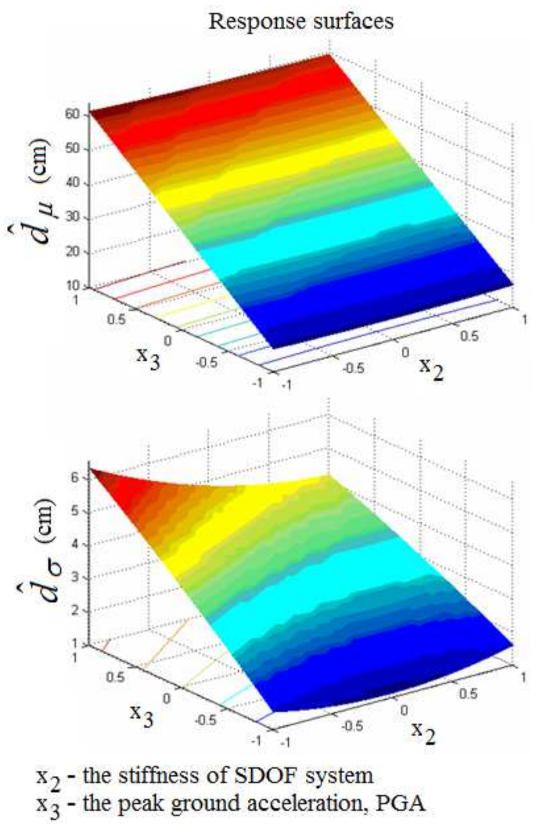


**Fig. 4.** Response surfaces for mean and standard deviation of the top displacement of SDOF system, as a function of stiffness and PGA

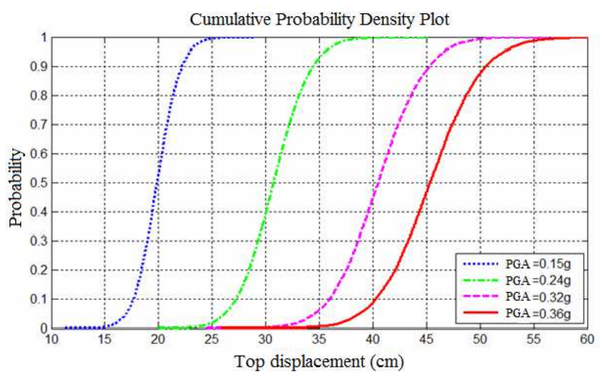


**Fig. 5.** Cumulative Probability Density of top displacement of SDOF system, for various values of PGA

The Cumulative Probability Density of top displacement of SDOF system, for various values of PGA is shown in Figure 5.

Assuming that the damage threshold of the top displacement is 20 centimeters, the damage probability of the SDOF system subjected to an earthquake having PGA=0.15g is 46%. At a damage threshold of 40 centimeters, the damage probability at a PGA value of 0.36g is 91% (Figure 5).

## 5. CONCLUSIONS

The Response Surface Methodology can be applied to a large variety of structures made of masonry, reinforced concrete, steel etc. It can be used with 2D as well as 3D models and the parameters representing the variables can be material characteristics, angle of seismic excitation at the base of the structure, the level of the seismic code, geometric characteristics.

The methodology presented in the paper is a very useful tool in assessing the fragility of structures. After obtaining the cumulative probability density and knowing the thresholds of different damage states, fragility curves can be easily derived.

## REFERENCES

1. Box, G. E. P., Wilson, K. B., *On the Experimental Attainment of Optimum Conditions*, Journal of the Royal Statistical Society, Series B, Vol. 13, Issue 1, pp. 1-45, 1951
2. Box, G. E. P., Draper, N. R., *Empirical Model-Building and Response Surfaces*, John Wiley & Sons, Inc., New York, 1986
3. Craig, J. I., Frost, J.D., Googno, B. J., Towashiraporn, P., Chawla, G., Seo, J. W., Duenas-Osorio, L., *Rapid Assessment of Fragilities for Collections of Buildings and Geostructures*, Project DS-5, Final Report, Mid America Earthquake Center, September, 2007
4. Myers, R. H., Montgomery, D. C., *Response Surface Methodology*, 2nd Edition, John Wiley & Sons, Inc., 2002