STRUT ELABORATION IN STRUT-AND-TIE MODEL

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ABSTRACT

The Strut-and-Tie Model (STM) has been incorporated into the codes and standards because of its consistency and rationality since last decade. However, it has encountered few challenges whilst its implementation. The effective compressive strength of strut has been a complex issue among researchers since emergence of STM. This review serves as a base for future developments of STM. It addresses several ways towards enhancing our understanding of strut performance in the STM. It also throws up questions for further investigation of effective compressive strength of strut in the STM. This review is confined to the evaluation of strut effectiveness factor based on the available codes and standards particularly AASHTO LRFD and ACI 318-08. According to the example given in this paper, there is sometimes a marked difference of approximately 50% between strut effectiveness factor recommended by AASHTO LRFD and ACI 318-08. More broadly, research is needed to determine which of the complicated strut effectiveness factor recommended by AASHTO LRFD and the simple one recommended by ACI 318-08 is optimum.

Keywords: discontinuity region; effective compressive strength; reinforced concrete; softening behaviour; strut-and-tie model

REZUMAT

Modelul grinzii cu zăbrele plastic (STM) a fost introdus în coduri, şi standarde, în ultimul deceniu datorită coerenţei şi raţionalităţii sale. Totuşi, modelul a trebuit să facă faţă unor critici, pe durata implementării sale. Capacitatea efectivă de rezistenţă la compresiune a bielei a constituit o problemă complexă pentru cercetători, încă de la apariţia STM. Recenzia de faţă constituie o bază pentru viitoare dezvoltări ale STM. Sunt prezentate diferite căi pentru îmbunătăţirea înţelegerii performanţei bielei din STM, formulând, totodată, întrebări pentru cercetări viitoare privind capacitatea efectivă de rezistenţă a bielei din STM. Recenzia este limitată la evaluarea factorului de eficienţă a bielei, pe baza codurilor şi standardelor disponibile, în mod deosebit a documentelor AASHTO LRFD şi ACI 318-08. Conform exemplului dat în articol, există uneori o diferenţă de cca. 50% între factorul de eficienţă a bielei, recomandat de AASHTO LRFD şi ACI 318-08. În sens mai larg, sunt necesare cercetări pentru a stabili care dintre factorii de eficienţă este optim: cel complicat, recomandat de AASHTO LRFD, sau cel simplu, recomandat de ACI 318-08.

Cuvinte cheie: zonă de discontinuitate; capacitate efectivă de rezistenţă la compresiune; beton armat; degradare de rigiditate, model de grindă cu zăbrele plastic.

1. INTRODUCTION

The Strut-and-Tie Model is a unified and rational approach which embodies a complicated structural member with a proper simplified truss model. It is commonly utilised to analyse the behaviour of discontinuity regions for structures members. Looking from another vantage point, it is a model for a portion of structure which represents a force system including balanced set of loads. Following the lower-bound theorem of plasticity, the factored member forces at each part of STM are confined to the corresponding design member strength [1, 2]. In 1899, the original truss model concept was initially recommended by Ritter to analyse shear problems [3, 4]. It was then developed for tension problems by Rausch in 1929 [5]. Later, the research on the STM was continued and several revised STM were recommended by researchers. In 2002, STM was recommended by ACI code rather than the simple equation which was used to predict the shear strength of reinforced concrete deep beams in previous versions of ACI code. Since last decade, there has been an increasingly growing body of literature published on STM [6-15]. Recent developments for design of deep concrete members such as pile cap and deep beam have heightened the need for using STM. However, many standards and codes have specified the STM for design and analysis of discontinuity regions for structure members [16-25].

Strut, as an important part of STM, is a region in which compressive stresses act parallel together from face to face of two nodes in the structure member. It is commonly idealised into three shapes of prismatic, bottle-shaped, and fan-shaped [16-18, 20-25]. The previous research findings show that, there is not unique strut dimension for one given concrete member to date. The rough estimate of strut dimensions is still an issue among researchers which has received some challenges for prediction of concrete strut behaviour in STM.

The crushing strength of concrete for strut is evaluated by effectiveness factor (ν). The available codes and standards which recommended strut effectiveness factor are classified into two groups in this review. The former group comprises AASHTO LRFD, CSA-S6-06, and CSA A23.3 that define the strut effectiveness factor as a function of the tensile strain of tie and angle between the strut and the tie [16, 19, 21]. The original idea of this effectiveness factor was proposed in 1986 by Vecchio and Collins [26]. The latter group comprises ACI 318-08, DIN 1045-1, NZS 3101, and CEB-FIP Model code 1999 that recommend a simple number as effectiveness factor depending on the type of concrete based on weight as well as the satisfaction of requirement reinforcements [17, 20, 22, 25].

The purpose of this paper is to review the available research about strut as the most important part of STM.

This review first gives a brief overview for definition of strut and its crack-control requirements according to the codes and standards. It then presents the strut dimensions and effectiveness factor equations which are the topics of interest in the present among researchers and structure designers. Finally, the effectiveness factors recommended by first and second group of codes and standards thereof are critically examined using a simple example.

2. DEFINITION OF STRUT

Simply, the region of the structure where the compressive force is acting is termed as strut. The same definition with replacing tensile force is used for the term of tie which commonly comes with strut. In the STM, the compressive stresses act approximately parallel to the direction of strut [1, 2, 6, 14]. The strut is commonly classified into three shapes of prismatic, bottle-shaped, and fan-shaped [16-20, 23-25]. In light of the concrete stress field is wider at mid-length of strut, the cross section of the strut generally varies along its length as illustrated in Figure 1 [2]. Thus, these types of strut are usually idealised as bottle-shaped as shown in Figure 1.

3. TRANSVERSE REINFORCEMENT REQUIREMENTS FOR STRUT

The transverse tension perpendicular to the strut centerline increases with the spreading of the compression forces which causes longitudinal cracks in the strut [1, 16, 19, 21, 23, 27]. Accordingly, the strut without transverse reinforcement fails due to the longitudinal cracks. Utilising adequate transverse reinforcement increases the strength of strut, which leads to the crushing failure of strut [1, 27]. Hence, many codes and standards, recommend the minimum requirement of transverse reinforcement to prevent immature cracking failure of strut [16, 17, 19, 21, 22, 24, 25]. Minimum requirement of reinforcement specified by ACI 318-08, AASHTO LRFD Bridge Design Specification, CSA-S6-06 (Canadian

Highway Bridge Design Code) is shown in Table 1 [16, 17, 19].

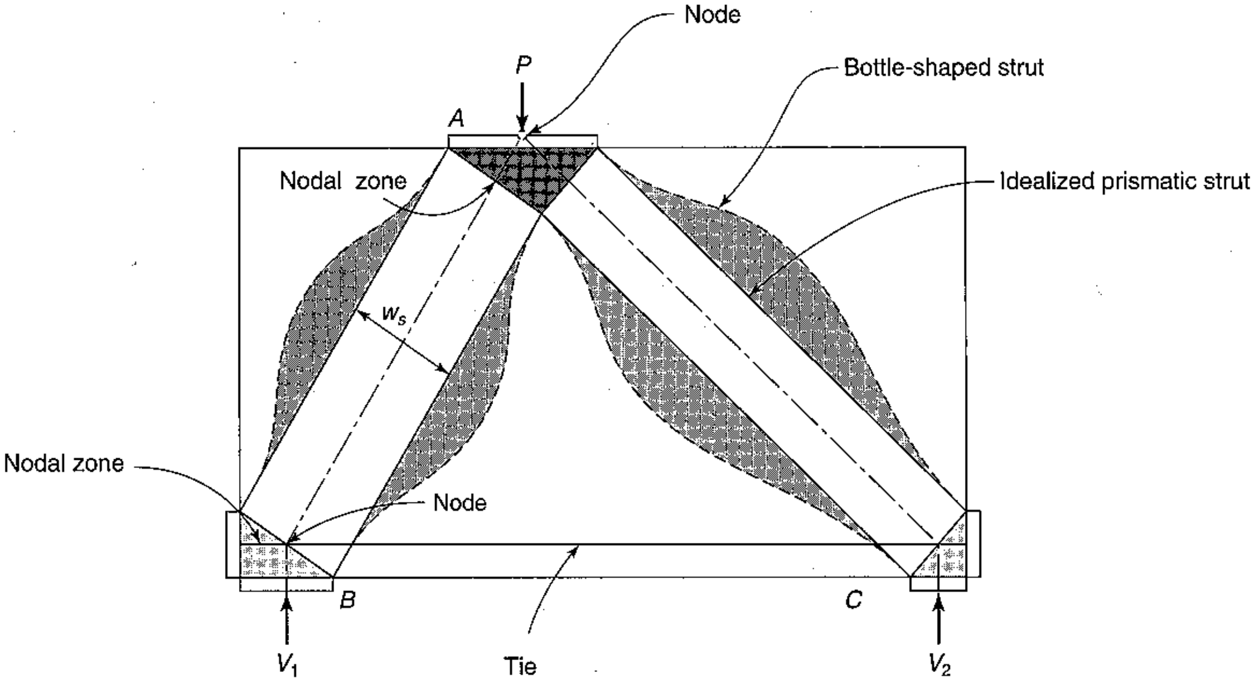

Fig. 1. Strut-and-tie model for a deep beam [2]

Table 1. Crack-control reinforcement across strut recommended by codes [2]

Specification	Minimum crack reinforcement across strut
AASHTO LRFD	Must have orthogonal grid of reinforcing bars near each face Spacing ≤ 12.0 in $\frac{\text{Cross section area of transverse reinforcement for each direction}}{\text{Gross concrete section area}} \geq 0.003$ (§ 5.6.3.6)
ACI 318-08	For ≤ 6000 psi $\sum \frac{A_{si}}{bs_i} \sin(\alpha_i) \geq 0.003$ (§ A.3.3.1)
CSA S6-06	Must have orthogonal grid of reinforcing bars near each face Spacing ≤ 300mm $\frac{\text{Cross section area of transverse reinforcement for each direction}}{\text{Gross concrete section area}} \geq 0.003$ Not more than 1500 mm2/m each face (§ 8.10.5.1)

However, increase of transverse reinforcement beyond these minimum requirements does not necessarily enhance the compressive strength of strut. The experiments indicate that at low ratio of shear span to effective depth of deep beam, the shear strength of beam and consequently compressive strength of strut decreases slightly with the increase of transverse reinforcement beyond the codes minimum requirements thereof, due to softening behaviour of concrete [1]. In last decade, using of new material such as Fibre Reinforced Polymer (FRP) to strengthen concrete deep members has become a topic of interest among researchers. Hence, the authors have undertaken the experiments to strengthen concrete strut and consequently increase RC deep beams shear strength utilising Carbon Fibre Reinforced Polymer (CFRP) sheet beyond the codes minimum transverse reinforcement requirements as shown in Figure 2. The experiment aims to develop the STM for RC deep beams strengthened by CFRP.

Fig. 2. Deep beam strengthening by CFRP using wet lay-up system

4. STRUT DIMENSIONS

The extent of strut in the out-of-plane dimension is equal to the thickness of structure [1, 3, 16, 17, 19-23, 27]. The critical dimension of strut stress area perpendicular to the strut centerline in the plane of structure varies from minimum value at two ends to maximum value at mid-length of bottle-shaped strut.

The dimension of bearing area at the end of bottle-shaped strut (*a*) as shown in Figure 3 is governed by the support conditions of strut, anchorage size, location of bearing plates as well as location and distribution of reinforcement. Some researchers assume that the bottle-shaped region extends approximately $1.5b_{ef}$ from one end of strut as shown in Figure 3 [2, 27, 28].

Besides that, the value of b_{ef} is assumed to be equal to maximum value between $l_s/3$ and *a* where l_s is the length of strut from face to face of the nodes [28]. However, some researchers point out the value of b_{ef} is often less than value of *a* for short struts. Thus, the equation (1) is assumed to revise the value of b_{ef} [2].

$$b_{ef} = \min\left((a + l_s / 6), b\right) \tag{1}$$

However, the findings from the previous research show that there is not unique strut dimension for one given concrete member. The rough estimation of strut dimensions is still an issue among researchers which has caused some challenges for prediction of concrete strut behaviour in STM.

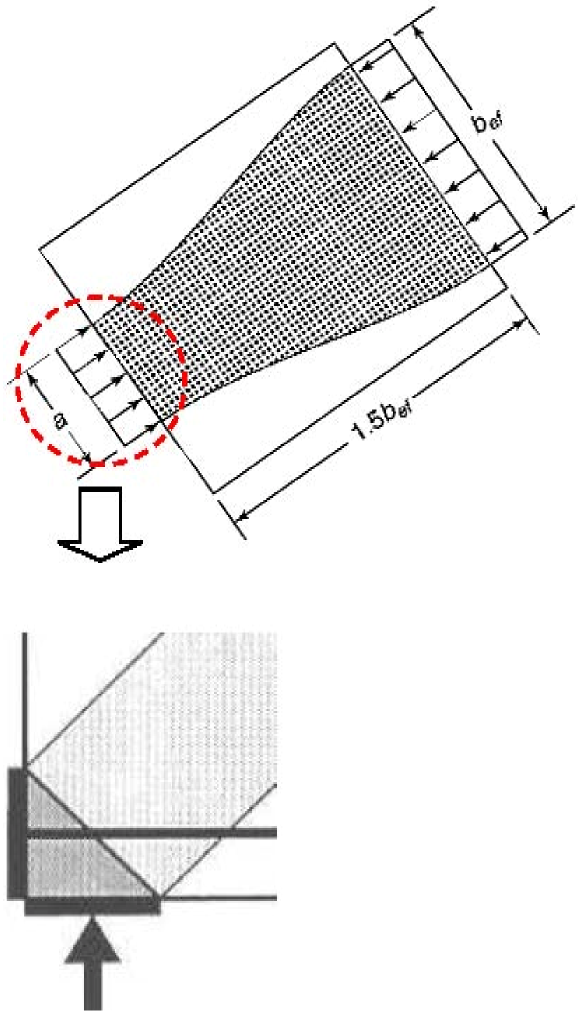

Fig. 3. Rough estimation of bottle-shaped strut dimensions with strut end details [2]

5. EFFECTIVE COMPRESSIVE STRENGTH OF STRUT

The crushing strength of concrete for strut is evaluated by effectiveness factor (ν). In general, the strut compressive strength parallel to its longitudinal axe is lower than uniaxial concrete compressive strength which is resulted from cylindrical test. The ultimate compressive stress of strut is termed effective concrete strength of strut (f_{ce}) and is calculated using equation (2).

$$f_{ce} = \nu f_c' \tag{2}$$

The effectiveness factor, which is also termed as reduction factor [18, 29] or efficiency factor [17] has the value between 0 and 1.0. Since two decades ago, numerous studies have been done to find the accurate value of strut effectiveness factor [27, 30-32]. Hence, different values are given for this factor in various codes and standards [16, 17, 19-25].

The main factors which affect the effective compressive strength of strut are concrete strength, load duration, tensile strain transverse to the strut, and concrete cracking [2]. According to these factors, prediction of effective compressive strength of strut varies among the codes and specifications.

Thus, based on the similarity of these code provisions, the effectiveness factor estimation is classified into two groups in this paper. The first group is comprised of AASHTO LRFD, CSA-S6-06, and CSA A23.3 that define the effectiveness factor of strut as a function of the angle between tie and strut as well as the tensile strain of tie.

Besides, the CSA A23.3 and the AASHTO LRFD confine the maximum value of effectiveness factor to upper limit of 0.85. The CSA-S6-06 allows this factor to be dropped to 0.67 depending on the concrete cylindrical concrete strength [16, 19, 21]. On the other hand, the second group is comprised of ACI 318-08, DIN 1045-1, NZS 3101, and CEB-FIP Model code 1999 that recommend a simple number as effectiveness factor depending on type of concrete based on weight and satisfaction of requirement reinforcements [17, 20, 22, 25].

6. COMPARISON OF EFFECTIVENESS FACTOR RECOMMENDED BY AASHTO LRFD AND ACI 318-08

Among available codes and standards, ACI 318-08 and AASHTO LRFD are chosen to discuss as a sample of each group thereof for their wide range of use. The recommendations of these codes for strut are illustrated in Table 2. There is sometimes a marked difference between the value of effectiveness factor recommended by AASHTO LRFD and ACI 318-08 depending on ε_1 and ε_s values. To clarify this difference, the node B in deep beam illustrated in Figure 1 will be considered. The strut along B-A direction is connected to the tie along B-C direction at node B.

According to AASHTO LRFD, ε_s is calculated by ratio of $f_y / 2E_s$. Thus, assuming the amounts of 440 MPa and 200 GPa respectively for f_y and E_s, ε_s will be equal to 0.0011 for the tie. The variation of effectiveness factor recommended by AASHTO LRFD and ACI 318-08 according to the different values of α_s from 25° to 65° is illustrated in Figure 4.

The variation of α_s is resulting from shear span to effective depth ratio variation from 0.5 to 2 recommended by codes and standards [16, 17, 19, 21-24].

$$\cot\alpha_s = \frac{a}{d} \tag{3}$$

Using equation (3), the values of effectiveness factor recommended by AASHTO LRFD and ACI 318-08 are indicated according to the variation of shear span to effective depth ratios (a/d) in Figure 5.

According to the graph from the AASHTO LRFD equation in Figure 5, with the increase of shear span to effective depth ratio from around 0.6 to 2 the effectiveness factor moderately decreases approximately from 0.84 to 0.32 respectively. It implies that the effective compressive strength of strut increases whilst the concrete beam become deeper according to the shear span to effective depth ratio from 2 to 0.5. This behaviour of strut is compatible with arch action. As shown in Figure 5, ACI 318-08 recommends the constant value of effectiveness factor for different shear span to effective depth ratios. Based on Figure 5, there is a marked difference between effectiveness factors recommended by codes thereof around +34.92 and -49.20 respectively for shear span to effective depth ratio of 0.5 and 2. Thus, this marked difference between effectiveness factor recommended by ACI 318-08 and AASHTO LRFD should be further explored in future research so that designers know whether to use the simple value of strut effectiveness factor recommended by ACI 318-08 or the complicated one recommended by AASHTO LRFD for the design of concrete deep members. Nonetheless, secondary struts play a crucial role in determining how the results are influenced by effectiveness factor.

Table 2. Effective compressive strength of strut specified by AASHTO LRFD and ACI 318-05 [16, 17]

Specification	Effective compressive stress of strut
AASHTO LRFD	$f_{ce}=\frac{f'_c}{0.8+170\varepsilon_1}\le 0.85f'_c$ $\varepsilon_1=\varepsilon_s+(\varepsilon_s+0.002)\cot^2\alpha_s$ (§ 5.6.3.3.3)
ACI 318-05	$f_{ce}=0.85\beta_s f'_c$ Prismatic: $\beta s=1.0$ Bottle-shaped with satisfying crack control: $\beta s=0.75$ Bottle-shaped without satisfying crack control: $\beta_s=0.60\lambda$ $\lambda=1.0$ for normal weight concrete $\lambda=0.85$ for sand-light weight concrete $\lambda=0.75$ for all lightweight concrete Strut in tension members: $\beta s=0.40$ All other cases: $\beta s=0.6$ (§ A.3)

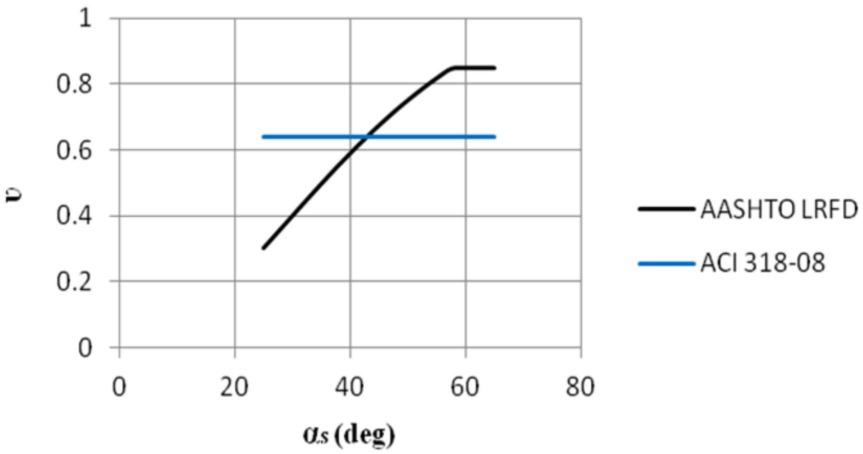

Fig. 4. Variation of the effectiveness factor based on the angle between tie and strut

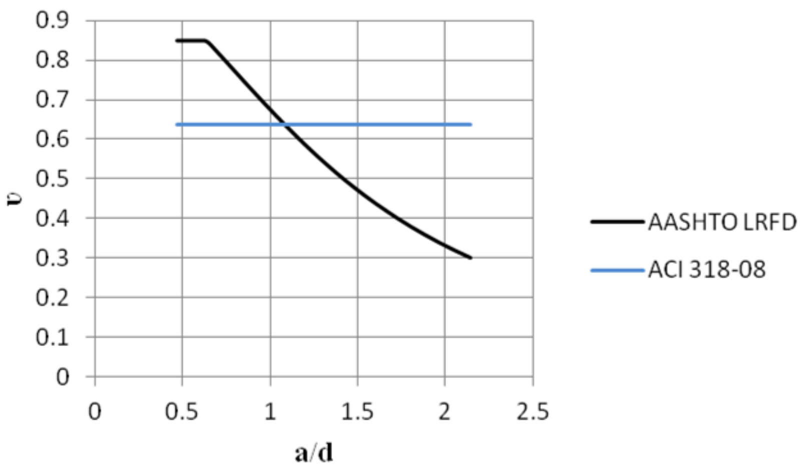

Fig. 5. Variation of the effectiveness factor based on shear span to effective depth ratio

7. CONCLUSIONS

This review sheds some light on the common challenges, which designers are encountering about strut whilst using strut-and-tie model.

The most significant findings emerged from this review are drawn as follows:

a) The increase of transverse reinforcement beyond the codes minimum requirements does not necessarily enhance the compressive strength of strut due to the softening behaviour of concrete.
b) There is sometimes a marked difference for the value of strut effectiveness factor between AASHTO LRFD and ACI 318-08 around 50%. Nonetheless, secondary struts play a crucial role in determining how the results are influenced by effectiveness factor.
c) There is no unique strut dimension for one given concrete member.
d) It would be interesting to assess the effectiveness factor of strut for high and ultra high strength concrete.

This study is confined to the evaluation of strut effectiveness factor based on available codes and standards particularly AASHTO LRFD and ACI 318-08. Based on this review, further work need to be done to clarify which of the complicated strut effectiveness factor recommended by AASHTO LRFD and the simple one recommended by ACI 318-08 is optimum.

NOMENCLATURE

a = effective width at the end of bottle-shaped strut (mm)

a' = shear span of deep beam (m)

A_{si} = crack control reinforcement cross section area adjacent to the two faces of the member with an angle of α_i to the crack (mm^2)

b = width of deep beam (mm)

b_{ef} = effective width at mid-length of bottle-shaped strut (mm)

d = effective depth of deep beam (m)

f_{ce} = effective compressive strength of strut (MPa)

f_y = yield stress of longitudinal steel bars (MPa)

f'_c = cylindrical compressive strength of concrete (MPa)

l_s = length of strut from face to face of the nodes (mm)

p = applied load for beam (N)

s_i = space among the orthogonal transverse reinforcements (mm)

v_1, v_2 = beam support reactions (N)

w_s = width of idealised prismatic strut (mm)

E_s = modulus of elasticity of steel bars (MPa)

β_s = coefficient recommended by ACI for effective strength of strut

λ = coefficient recommended by ACI to calculate β_s

ε_1 = transverse strain of concrete strut perpendicular to its centerline (mm/mm)

ε_s = tensile strain in the direction of a tie (mm/mm)

α_i = angle at each layer of reinforcement crosses strut (rad)

ν = strut effectiveness factor

α_s = angle between strut centerline and tie (rad)

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