SEISMIC HAZARD VERSUS DESIGN ACCELEROGRAMS

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ABSTRACT

The developments presented herewith arise from the legitimate wish and task of structural engineers of performing consistent analyses of risk and safety for works located in seismic regions. The paper adopts a probabilistic viewpoint, or philosophy. The starting point of this discussion is based on well – established approaches, while at the same time addressing specific research problems that could represent the task of the future, even if not an immediate one. The main topics are as follows: a brief view of (3-rd level) probabilistic safety and risk analysis, based on simplifying assumptions, which serves as a starting point for subsequent developments; a review of seismic action and hazard representation from two perspectives: ground motion during one event (advocating design accelerograms and stochastic models) and recurrence of successive events (considering, after the usual 1D approach, a generalized approach based on the consideration of a multi-dimensional space of characteristic action parameters); the development of a multi-dimensional stochastic model of ground motion (basically for a half-space consisting of successive horizontal homogeneous layers); a brief discussion of alternative possible objectives of engineering safety analyses.

Keywords: seismic action, seismic hazard, seismic risk, stochastic models, design strategies

REZUMAT

Dezvoltările prezentate sunt determinate de dorința legitimă şi misiunea inginerilor structurişti de a efectua analize coerente de risc şi siguranță pentru lucrările amplasate în zone seismice. Lucrarea are la bază o filosofie probabilistică. Punctul de plecare al acestei discuții are la bază abordări larg acceptate, abordând de asemenea unele probleme de cercetare specifice, care ar putea să reprezinte o misiune de viitor, chiar dacă nu a unui viitor foarte apropiat. Principalele teme abordate sunt: o scurtă trecere în revistă a analizei probabilistice (de nivel 3) pentru siguranță şi risc, bazată pe unele ipoteze simplificatoare, care serveşte ca punct de plecare pentru dezvoltările următoare; o privire asupra acțiunii seismice şi hazardului corespunzător, din două perspective: mişcarea terenului în timpul unui eveniment (argumentându-se în favoarea utilizării accelerogramelor de proiectare şi a modelelor stochastice pentru proiectare) şi recurența evenimentelor succesive (considerându-se, după uzuala abordare monodimensională, o abordare generalizată, bazată pe considerarea unui spațiu multidimensional al parametrilor caracteristici ai acțiunii); dezvoltarea unui model stochastic multidimensional al mişcării terenului (în esență, pentru un semispațiu cu o stratificație plan-paralelă de corpuri deformabile omogene); o scurtă discuție a unor posibile obiective ale analizelor inginereşti ale siguranței.

Cuvinte cheie: acțiune seismică, hazard seismic, risc seismic, modele stochastice, strategii de proiectare

1. INTRODUCTION

This is a discussion and opinion paper, intended to deal with the needs and requirements of engineers involved in earthquake protection of various works and facilities (mostly, civil engineering structures). The basic goal of engineering activities is to develop solutions for new works or to evaluate (in view of possible subsequent intervention) existing structures for potential rehabilitation / upgrading. It is most desirable to conduct such activities under conditions of control of safety or, conversely, of risk, of occurrence of adverse effects of earthquakes. The developments herewith presented arise from the legitimate wish and task of structural engineers to perform consistent analyses of risk and safety for works located in seismic regions. While some of the topics under discussion may appear quite common, several challenges to the current state of the art are also raised, and part of them may be a matter of activities or trends of a rather more remote future.

The philosophy adopted in the paper relies on a probabilistic approach to the anticipation of events concerning seismic hazard and risk. The author believes that the knowledge available to date does not offer a logical, consistent, alternative to the acceptance in this field of a probabilistic philosophy. Quite classical models and approaches are used in this respect, but these are followed also by some extensions that are not usual. On the other hand, it is implicitly assumed that relying on an appropriate, consistent, basic deterministic modelling of physical phenomena remains an unavoidable prerequisite.

It is well commonly accepted that codes for practice, in the field of structural design, including earthquake resistant design (as well as other technical fields), are developed on the basis of simplifications determined on one hand by the limits to knowledge and on the other hand by the limits to analysis and design efforts that can be asked for in practical activities. The field of earthquake engineering represents a strong illustration in this respect. A look at the past reveals a steady and multi-faceted progress at the forefront of knowledge in this field. It also reveals the gradual implementation of this new knowledge in technical codes, while an unavoidable gap nevertheless persists.

The paper emphasizes implicitly some basic inconsistencies of the codes of structural design (essentially in relation to earthquake resistant design problems) and intends to contribute to the gradual surpassing of such inconsistencies. Earthquake resistant design ranks high from the point of view of several relevant criteria, including the basic information required, the complexity of analyst's tasks, or the social stake of investment requirements, but above all of the high stakes of safety and reliability of the outcome of design. Therefore, earthquake resistant design can be considered to a large extent a pioneering activity in the general field of structural design. Problems raised and new approaches that are developed in this field often become a source of ideas for other fields too.

The main theme of the paper is a discussion and research contributions on following:

1. Review of some main features of engineering safety analyses.
2. Representations related to seismic action and hazard.
3. Analytical developments related to ground motion models.
4. Objectives / strategies of engineering calculations involved by design activities.

2. SOME REFERENCES TO STRUCTURAL SAFETY ANALYSES

A formal, schematic, framework of analysis of seismic risk affecting structures, is used at this point as a reference for more specific developments, as referred to in subsequent sections. A probabilistic approach is used. The approach presented is the so-called 3-rd level approach, to distinguish it from the 1-st level approach on which current codes for practice (e.g. the Eurocode *EC8* (CEN, 2004)) rely, or from the 2-nd level approach (e.g. FOSM), based on the use of lower order moments of random variables involved. It may be worth mentioning in this respect that only the 3-rd level approach, which is currently not yet a tool for practice, allows for consistent, in depth, safety / risk analyses with a suitable flexibility of objectives.

The basic model, obviously strongly idealized (1D approach, discrete modelling etc.) is as follows:

a) *with respect to seismic action and hazard*:
   - it is assumed that seismic action may occur, as a sequence of events of negligible individual duration,

at various possible time moments t_i, during a certain reference (long) time interval of duration T;

- it is assumed that seismic action (at the level of a definite site) may occur with various severity characteristics, which may be globally quantified by means of a scalar parameter q, which is quantified subsequently at its turn by means of discrete values q_j, (where the indices j = 1... J are positive, integer values); moreover, it is assumed that the sequence of values q_j represents levels of severity increasing with increasing j;
- the likelihood of occurrence of seismic action (at a sequence of randomly occurring levels of severity that may endanger various exposed elements), is referred to as *seismic hazard* (at a site of interest dealt with) and is quantified according to subsequent developments;
- the seismicity at site level, or more precisely local seismic hazard, referred to at a level that is relevant for a site of interest, is assumed to be stationary and to correspond to a Poissonian model, which is basically characterized by the sequence of values $n^{(h)-}_j$ (where the superscript $^{(h)}$ stands for *hazard*), representing (in probabilistic terms) the *expected frequencies of occurrence* of events of severity levels q_j; besides this, one may consider the *expected cumulated frequencies* of reaching or exceeding the values q_j, $N^{(h)-}_j$,

$$N^{(h)-}_j = \Sigma_{j'}^{j,J}\, n^{(h)-}_{j'}, \qquad (2.1)$$

the *expected return periods* $T^{(h)-}_j$ of events having levels of severity not less than q_j, i.e. $q_{j'} \in (q_j, q_J)$,

$$T^{(h)-}_j = (N^{(h)-}_j)^{-1}, \qquad (2.2)$$

the *probabilities of non – occurrence and non – exceedance, during an observation time interval* T, of a level of severity q_j, $\boldsymbol{P}^{(h)}_{j0}(T)$ (where bold characters stand in this subsection for probabilities), which according to the Poissonian model are

$$\boldsymbol{P}^{(h)}_{j0}(T) = \exp(-T / T^{(h)-}_j), \qquad (2.3)$$

more generally, the *probabilities of m times occurremce or exceedance* (m = 0, 1, 2, ...), *during an observation time interval T*, of a level of severity q_j, $\boldsymbol{P}^{(h)}_{jm}(T)$, which according to the Poissonian model are

$$\boldsymbol{P}^{(h)}_{jm}(T) = \exp(-T / T^{(h)-}_j) \times (T / T^{(h)-}_j)^m / m! \qquad (2.3')$$

(of course, $\Sigma_m^{0,\infty}\ \boldsymbol{P}^{(h)}_{jm}(T) \equiv 1$);

b) *with respect to a structure dealt with*:

- it is assumed that the structure may be affected, due to earthquake occurrence, at various levels of damage, which may be globally quantified by means of a scalar parameter d, which may take discrete values d_k, (where the indices k = 0... K are non-negative, integer values); moreover, it is assumed that the sequence of values d_k represents levels of damage severity increasing with increasing k, where k = 0 means intact structure, while $k = K$ means maximum possible damage, or destruction;
- it is assumed that, after every seismic event, the structure is promptly and perfectly rehabilitated, such that successive seismic events encounter in each case of occurence, the same structure;

- the possible effect of occurrence of seismic action at a level of severity q_j will be that, that damage of a level of severity d_k may affect the structure, i.e. that the structure is vulnerable; the damage severity level d_k is random; the *vulnerability of a structure dealt with* will be characterized by the system of conditional probabilities $\boldsymbol{p}^{(v)}{}_{k/j}$ (where the superscript $^{(v)}$ stands for vulnerability); (of course, $\Sigma_k^{0,K}\, \boldsymbol{p}^{(v)}{}_{k\,/\,j} \equiv 1$, for any index j);

c) *with respect to the risk of structures to be affected*:

- it is assumed that earthquake induced damage may occur, as a sequence of events at various possible time moments t_i, during a certain reference time interval T, due to the occurrence of earthquakes and to the existence of seismic vulnerabililty of structures; the sequence of these (adverse) effects is characterized and quantified according to the features of hazard and vulnerability, referred to previously;
- the likelihood of occurrence of earthquake induced damage (at randomly occurring levels of severity), is referred to as *seismic risk* (for a structure dealt with) and is dealt with according to subsequent developments;
- the seismic risk is assumed to be stationary and to correspond to a Poissonian model, which is basically characterized by the sequence of values $n^{(r)-}{}_k$ (where the superscript $^{(r)}$ stands for *risk*), representing (in probabilistic terms) the *expected frequencies of occurrence* of damage of severities d_k; the values $n^{(r)-}{}_k$ are determined on the basis of the convolution expression

$$n^{(r)-}{}_k = \Sigma_j^{o,J}\, \boldsymbol{p}^{(v)}{}_{k\,/\,j} \times n^{(h)-}{}_j, \qquad (2.4)$$

- besides this, one may consider the *expected cumulated frequencies* $N^{(r)-}{}_k$, the *expected return periods* $T^{(r)-}{}_k$ of events having levels of severity not less than d_k, the *probabilities of non – occurrence and non – exceedance, during an observation time interval T*, of a level of severity d_k, $\boldsymbol{P}^{(r)}{}_{k0}\ (T)$, or more generally, the *probabilities of m times occurrence or exceedance* (m = 0, 1, 2, ...), *during an observation time interval T*, of a level of severity d_k, $\boldsymbol{P}^{(r)}{}_{km}\ (T)$, on the basis of expressions that are homologous to the expressions (2.1)...(2.3').

It must be mentioned that the previous developments are rather illustrative and correspond to the simplest possible situation, or models. The various assumptions accepted may be questioned and may be generalized, which would lead, of course, to more complicated ways of quantification and relations. Some of the possible generalizations, which concern seismic hazard, are dealt with in subsequent section. On the other hand, it is worth mentioning that the rules of safety verification specified by codes for current practice (e.g.: (CEN, 2004)) are not (yet) based on convolutions of type (2.4), but use conventional (design) values related to fractiles of distributions of parameters characterizing hazard and vulnerability.

3. CONSIDERATIONS AND DEVELOPMENTS ON THE SPECIFICATION OF SEISMIC ACTION AND HAZARD FOR DESIGN

3.1. General

The following developments concern a formal approach to some basic aspects of characterization and quantification of ground motion and seismic hazard. Two main aspects are considered at this point, from the point of view of the requirements of engineering activities:

a’) representation related to seismic action during one event, or case of occurrence;

b’) representation related to the sequence of cases of occurrence of seismic action.

The two aspects referred to are related to two different time scales (or orders of magnitude of time intervals considered), which can be dealt with separately, due to the fact that earthquakes are transient phenomena of a duration usually not exceeding the order of magnitude of about one minute, while the sequences of successive earthquakes are to be followed usually for time intervals in the range of centuries, if not even longer.

The representations referred to are of course different, but they are also interrelated, since the representation (b’) will rely on the solution adopted for the representation (a’). The normal succession of analysis is thus that of dealing first with the aspect (a’) and thereafter with the aspect (b’).

In another respect, it is most important to consider the conceptual difference between:

a”) the events, as well as the sequence of events, observed in the past;

b”) the events and the sequence of events to occur in future.

While for the entities of category (a”) there should exist in principle hard information (desirably of instrumental nature), which permits deterministic basic analyses, for the entities of category (b”) there appears a need of anticipation, for which, according to present concepts, the use of probabilistic approaches (if feasible, due to limits to basic information at hand) represents the most consistent solution. Design represents an activity which is bound to rely on concepts and approaches corresponding to this latter category of entities.

The developments of the paper are related, in their turn, to aspects that are specific to the consideration of category (b”) entities and specific problems.

3.2. The case of an individual event

3.2.1. Alternative representations of time dependence during one event

It may be stated that the representations / characterizations used most frequently for predictive purposes in engineering analysis activities pertain to three different categories:

a) *R.Sp.*: design spectrum representation;

b) *R.Ac.*: design accelerogram representation;

c) *R.St.*: stochastic design representation.

The representation *R.Sp.* is inherently related to a single component (or DOF) of ground motion (and, consequently, of the motion of the ground – structure interface). So, this representation can provide just some basic information on the amplitude and the spectral content of *one* (usually horizontal translation) ground motion component, but it is definitely non-satisfactory for the consideration and characterization of the simultaneous motion along the different degrees of freedom of the interface. By the way, it may be mentioned that the needs and ways to deal with non-synchronous seismic input in structural analysis were dealt with e.g. in (Clough & Penzien, 1975) or (Sandi, 1970, 1983). It is also to be mentioned that the use of the representation *R.Sp.* is consistent basically with the semi-probabilistic (or probabilistic of level I) approach to risk and safety problems (which is currently adopted in codes and used in practice, but is of questionable relevance for safety estimates).

The representation *R.Ac.* can be related to the motion along an *arbitrary number of degrees of freedom*. It also presents the great advantage that it can be considered and used in relation to the analysis of linear or non-linear behaviour of structures. There is nevertheless also another problem: even if one forgets about the need of consideration of the random, unpredictable (in deterministic terms) nature of ground motion and one would agree to use past, recorded accelerograms (which will indeed never occur again as such): there are never satisfactory systems of actual records available (records which should provide information on the motion precisely along *all relevant degrees of freedom of the ground–structure interface of a structure of interest*). Therefore, the use of the representation *R.Ac.* *requires a logical support of anticipative nature*, and this can be provided by an appropriate basic model, required to have at its turn an appropriate logical support. This support may become available, according to the author's views, only by the consideration of the representation *R.St.*, referred to before, which is dealt with further on as a basic representation. The consequence is that the design accelerograms should be generated in a way to fit the requirements of Monte-Carlo type analyses.

Concerning the representation *R.St.*, it is worth mentioning that earthquake ground motions of interest in earthquake engineering activities are widely accepted to be random, and are also *inherently transient, i.e. non-stationary.* On the contrary, in the frame of the attempts to use this representation in codes, like e.g. in some successive drafts of the Eurocode *EC8* (CEN, 2004), the stochastic models referred to are *implicitly stationary*. A position versus this situation is required. In a different connection, the representation *R.St.* can be considered in connection with an arbitrary number of degrees of freedom of the ground - structure interface (provided corresponding models are available). On the other hand, it turns out that the representation *R.St.* is well suited from the viewpoint of practical feasibility for linear analyses only, while attempts to use it for non-linear analyses encounter in most cases insurmountable difficulties (consider e.g. the case of non-linear constitutive laws for structural components, which should account for multi-dimensional hysteretic / degrading characteristics).

Note also that the representations *R.Ac.* and *R.St.* fit, if safety or risk analyses are intended, rather to the consistent probabilistic approach than to the semi-probabilistic one. The approach advocated previously corresponds to the desire and goal of performing consistent analyses of vulnerability and even of risk, in agreement with the developments of previous section and oriented along the strategies *S*.3 or *S*.4, defined in Section 5. Of course, a pragmatic approach, which is by far less demanding, can accept and even recommend the use of natural (recorded) accelerograms, selected according to some quite simple criteria. Without denying the value of pragmatic orientations, it may be stated that they cannot lead to explicit control of safety and risk.

Given these facts, it appears that a basic strategy in developing anticipative ground motion representations intended for rather consistent engineering analyses should consist of:

- adoption of a suitable stochastic model of ground motion of category *R.St.* (or, forgetting for the moment about ground-structure interaction, of ground-structure interface motion),
- generation and use in engineering analyses of (systems of) design accelerograms of category *R.Ac.*, derived accordingly to a basic model of category *R.St.* (an example in this sense is provided by the developments of (Bălan & al., 1977), concerning a multi-support structure).

In order to deal with random seismic ground motion, non-stationary random functions should be considered. The random functions referred to, which are to be specific to a definite type of structure (more precisely, of ground – structure interface), should be in principle vectorial, since a consistent analysis of structural performance requires consideration of the simultaneous motion along the various degrees of freedom of the ground - structure interface. The direct analysis in such a frame is nevertheless difficult, first of all because of the lack of data concerning the (two argument) auto-correlations and cross-correlations which are basic for the characterization of such functions. To get a more feasible way, the advantages of dealing with stationary random functions should be recalled. As a start point, the canonic expansion (Pugachov, 1960) of a random, non - stationary, vectorial, ground accelerogram $\boldsymbol{w}_g^{(n)}(t)$ (in subsequent developments, vectors are represented by lower case bold letters, while matrices are represented by upper case bold letters) is

$$\boldsymbol{w}_g^{(n)}(t) = \Sigma_k\, f_k(t) \times \boldsymbol{w}_g^{(s)}{}_k(t) \qquad (3.1)$$

whereby the stationary (vectorial) random functions $\boldsymbol{w}_g^{(s)}{}_k(t)$ are assumed not to be cross-correlated, while $f_k(t)$ represent scalar envelopes accounting for the non - stationarity of motion. Note here that the expansion of relation (3.1) can be usually rather well correlated with the sequence of successive trains of seismic waves (*P*-waves, *S*-waves etc.), so it may have a quite attractive physical sense. In case one uses as a start point the expansion (3.1), the problem of anticipating the accelerograms implies a specification in appropriate terms of the two factors of the right member.

The random functions representing (scalar or vectorial) accelerograms can be fully characterized in this case by their auto-correlation (or covariance) functions or by their Fourier transforms, the ("power") spectrum densities. One can refer in this connection to three variants:

a) the case of consideration of stationary random functions like $\boldsymbol{w}_g^{(s)}{}_k(t)$, for which the classical Wiener – Khinchin relations (Pugachov, 1960) (reproduced subsequently) are valid;

b) the case of consideration of non - stationary random functions like $\boldsymbol{w}_g^{(n)}(t)$, for which the generalized Wiener – Khinchin relations (Crandall, 1963) are valid;

c) the case of consideration of non-stationary random functions like $\boldsymbol{w}_g^{(n)}(t)$, for which the "diagonal" time and frequency arguments are used and for which the correspondingly adapted generalized Wiener – Khinchin relations (Sandi, 1989) are valid (note here that this latter variant eventually permits a smooth passage from the representations of variant (b) to those of variant (a)).

Given the fact that the use of stationary random functions like $\boldsymbol{w}_g^{(s)}{}_k(t)$ is well suited for the generation of artificial accelerograms, the canonic expansion of expression (3.1), in connection with the classical Wiener – Khinchin relations, will be considered as a basis further on.

The auto-correlation (matrix) function $\boldsymbol{B}\,[\boldsymbol{w}_g^{(s)}{}_k(t);\, t_n]$ of a stationary (vectorial) function is defined as

$$\boldsymbol{B}\,[\boldsymbol{w}_g^{(s)}{}_k(t);\, t_n] = \\ = <\boldsymbol{w}_g^{(s)}{}_k(t_1) \times \boldsymbol{w}_g^{(s}{}_k{}^{T}(t_2)> \qquad (3.2)$$

where: $t_n = t_2 - t_1$, $t_m = (t_1 + t_2)/2$ (the latter: dummy, in case of stationary functions), as introduced in (Sandi, 1989) for the "diagonal" representation; the superscript $^{“T”}$ means the transpose of the vector; the symbol < … > means averaging upon the statistical ensemble, which can be replaced, for ergodic functions, by averaging in time upon one single sample function; and "×" means the dyadic product of the two vectors considered.

The classical Wiener – Khinchin relations concerning the stationary random vectors $\boldsymbol{w}_g^{(s)}{}_k(t)$ of expression (3.2) are written as

$$\boldsymbol{S}[\boldsymbol{w}_g^{(s)}{}_k(t);\ \omega_m] = (1\,/\,2\pi) \int_{-\infty}^{\infty} \exp(-\mathrm{i}\omega_m\, t_n)\, \boldsymbol{B}[\boldsymbol{w}_g^{(s)}{}_k(t);\, t_n]\, \mathrm{d}t_n \qquad (3.3)$$

$$\boldsymbol{B}[\boldsymbol{w}_g^{(s)}{}_k(t);\, t_n] = \int_{-\infty}^{\infty} \exp(\mathrm{i}\omega_m\, t_n)\, \boldsymbol{S}[\boldsymbol{w}_g^{(s)}{}_k(t);\ \omega_m]\, \mathrm{d}\omega_m \qquad (3.4)$$

3.3. Spaces of seismic action and representation of seismic hazard. 1D approach.

In the simplest approaches (which unfortunately apply to the design codes currently in force) the space of characteristics of seismic action for which randomness is explicitly accounted for is mono – dimensional (1D). The corresponding coordinate *q* (representing possible values of a continuous random, scalar, parameter *Q* used hereafter) could mean seismic intensity, or peak ground acceleration, or peak spectral velocity, or some reference amplitude etc.. Some basic parameters and relations concerning this case are given subsequently, as an element of reference and comparison for the case of a generalized, *n*D, approach, dealt with thereafter. In the 1D case one can consider following main characteristics (if a Poissonian recurrence model is accepted):

- an *expected occurrence frequency density,* denoted $n^{(h)}{}_Q\ (q)$ and an *expected occurrence frequency* of cases of values $Q \geq q$, denoted $N^{(h)}{}_Q\ (q)$, where

$$N^{(h)}{}_Q\,(q) = \int_q^\infty n^{(h)}{}_Q\,(q')\,\mathrm{d}q', \qquad (3.5)$$

- an *expected occurrence return period,* denoted $T^{(h)}{}_Q\,(q)$, given by the expression

$$T^{(h)}{}_Q\,(q) = [\int_q^\infty n^{(h)}{}_Q\,(q')\,\mathrm{d}q']^{-1} = = [N^{(h)}{}_Q\,(q)]^{-1}, \qquad (3.6)$$

- probabilities of non – exceedance, $P^{(h)}{}_{Q0}\ (q,\ T)$, or of *m*-time exceedance, $P^{(h)}{}_{Qm}\,(q,\ T)$ etc., where e.g.

$$P^{(h)}{}_{Q0}\,(q,\ T) = \exp\,[-\ T\ N^{(h)}{}_Q\,(q)], \qquad (3.7)$$

$$P^{(h)}{}_{Qm}\,(q,\ T) = = \exp\,[-\ T\ N^{(h)}{}_Q\,(q)] \times [N^{(h)}{}_Q\,(q)]^m\ /\ m!, \qquad (3.8)$$

with the obvious condition

$$\Sigma_m{}^{0,\infty}\ P^{(h)}{}_{Qm}\,(q,\ T) \equiv 1.$$

3.4. Spaces of seismic action and representation of seismic hazard. nD approach.

A look at the wealth of actual accelerographic records reveals a multi-dimensional variability (related to features like amplitude, dominant frequency, duration…) of ground motion characteristics. At least currently, such characteristics are not predictable in deterministic terms, and a recognition of their randomness, followed by the use of probabilistic tools for corresponding analyses, appears to be a logical solution in order to predict, in a way that is relevant for engineering activities, the features of ground motions to occur. Therefore, a multi-dimensional space of ground motion characteristics should be introduced in order to perform predictive analyses. Due to pragmatic reasons, the adoption of an *n*-dimensional space, having a reasonable, limited, number *n* of dimensions, corresponding to the most significant parameters characterizing ground motion, appears to be a suitable solution. Some stark compromises in this choice are, of course, unavoidable, first of all due to the fact that, in principle, the dimension *n* of this space should cover the product of (the number of DOF of the ground – structure interface, or of components of the vector $\boldsymbol{w}_g{}^{(n)}(t)$), times (the number of selected characteristics of motion along one component of the vector $\boldsymbol{w}_g{}^{(n)}(t)$), times (the number of characteristics of the relationship between different components). A look at relations (3.1) … (3.4) could be useful in deciding upon the selection required. To come closer to reality and practice, one could introduce a space $\boldsymbol{S_L}$ of a finite number of dimensions, having the coordinates q_l (l = 1 … L), and a possible way to do this is represented by the use of some techniques of discretization of the basic characteristics $f_k(t)$ of relation (3.1) and $\boldsymbol{S}[\boldsymbol{w}_g{}^{(s)}{}_k(t);\ \omega_m]$ of relations (3.3) and (3.4). This could be done, e.g., by introducing a stepwise variation of the envelope $f_k(t)$ with respect to the time coordinate t, and of the classical spectrum density matrix $\boldsymbol{S}[\boldsymbol{w}_g{}^{(s)}{}_k(t);\ \omega_m]$ with respect to the (average) circular frequency ω_m.

In spite of the difficulties raised by this option, some basic relations are presented subsequently. Their consideration, even if seldom practical, makes possible a critical look at the outcome of use of relations (3.5) … (3.8) presented previously. The basic implications of the use of an *n*D space instead of the 1D space considered in subsection 3.3 are:

- replacing of the scalar q by a vector $\boldsymbol{q}$ of components q_l (l = 1 … L) and the random scalar Q by a corresponding vector $\boldsymbol{Q}$;

- replacing conditions $Q < q$ by conditions $(q_l) \in \Omega$, where Ω is a domain of the space $\boldsymbol{S_L}$.

In this latter variant the expected frequency density $n^{(h)}{}_Q\,(q)$ of relations (3.5) and (3.6) would be replaced by an expected frequency density $n^{(h)}{}_{\boldsymbol{Q}}\,(q_l)$, where q_l are the coordinates of the space, while the expected frequency $N^{(h)}{}_Q\,(q)$ would be replaced by an expected frequency of cases satisfying the condition $(q_l) \in \Omega$, denoted $N^{(h)}{}_{\boldsymbol{Q}}\,(\Omega)$, where Ω is a domain of the space of coordinates q_j. Instead of the expression (3.5) one has now

$$N^{(h)}{}_{\boldsymbol{Q}}\,(\Omega) = \int_\Omega n^{(h)}{}_{\boldsymbol{Q}}\,(q_l)\, \mathrm{d}\Omega, \tag{3.9}$$

while instead of the expression (3.6) one has, for the return period of occurrence of events with $(q_l) \in \Omega$,

$$T^{(h)}{}_{\boldsymbol{Q}}\,(\Omega) = [\int_\Omega n^{(h)}{}_{\boldsymbol{Q}}\,(q_j)\, \mathrm{d}\Omega]^{-1}. \tag{3.10}$$

The expressions (3.7) and (3.8) will be replaced in this case by

$$P^{(h)}{}_{\boldsymbol{Q}0}\,(\Omega, T) = \exp\,[-\,T\, N^{(h)}{}_{\boldsymbol{Q}}\,(\Omega)] \tag{3.11}$$

and

$$P^{(h)}{}_{\boldsymbol{Q}m}\,(\Omega, T) = \\ = \exp\,[-\,T\, N^{(h)}{}_{\boldsymbol{Q}}\,(\Omega)] \times [N^{(h)}{}_{\boldsymbol{Q}}\,(\Omega)]^m / m!. \tag{3.12}$$

4. A STOCHASTIC GROUND MOTION MODEL

4.1. General

This section is devoted to an analytical approach to the development of a stochastic model of earthquake ground motion, while ground is represented as a 3D continuum. The motion is non-synchronous at different points and along different directions, due to the features of wave propagation, first of all due to the finite wave propoagation velocities. The model presented is related to stationary ground motion, but it can provide a basis for modelling non-stationary ground motion too, on the basis of the canonic expansion (3.1).

Making a comparison with approaches of more pragmatic nature, dealing also with case studies, like those of (Frankel, 1994), (Moriwaki & al., 1994) or (Spudich, 1994), where specific seismological aspects are tackled and a probabilistic view point is accepted and partially used, it turns out that the subsequent formal developments are more comprehensive, being intended to build a bridge towards consistent safety and risk analysis. They could suggest, of course, also some ways of gradual refinement of pragmatic aprroaches, in order to make them more relevant for safety control. It may be also stated that the analytical developments are complementary to in depth case studies, like e.g. that of (Frankel, 1994), which can provide valuable factual experience, but require also some control / completion based on more comprehensive modelling.

A brief review of the main questions to which the specification of seismic design input should answer could be presented as follows:

a) how to select the degrees of freedom of the ground – structure interface which are to be considered in specifying the seismic input?
b) how to represent the (expected) time dependence of input accelerations of future events?
c) how to consider and represent in an anticipative manner the simultaneous (and non–synchronous) time dependence of input along the different degrees of freedom of the ground–structure interface?
d) what kind of expected recurrence model for the sequence of future seismic events to adopt?
e) how to correlate the aspects referred to previously with the use of the philosophy of performance based design?
f) what kind of position to adopt versus the problem of risk analysis? how to select the objective(s) of engineering analyses?

The previous items represent obviously an extensive shopping list, which cannot be dealt with thoroughly in the frame of this paper. Given the profile of this section, the aspects to which the developments presented pertain concern essentially the items (b) and (c). The item (f) is dealt with in Section 5. An answer to these questions is as follows: the option is for a stochastic model, developed in agreement with the canonic expansion (3.1), for which the detailed specification corresponds to the spectrum density matrix $\boldsymbol{S}[\boldsymbol{w}_g{}^{(s)}{}_k(t);\ \omega_m]$ used in the classical Wiener – Khinchin relations (3.3), (3.4).

The consideration of the randomness of seismic ground motion has become quite common in several engineering activities. Stochastic ground motion models are by now explicitly or implicitly accepted, in various forms, in some regulations, as e.g. the various parts and draft editions of the Eurocode *EC8* (CEN, 2004),

In order to deal with random seismic ground motion, non - stationary random functions should be considered. The direct analysis in such a frame is nevertheless difficult, first of all because of the lack of data concerning the (two argument) auto-correlations and cross-correlations of such functions, representing the motion along the various degrees of freedom of ground-structure interfaces. For an approach that is more feasible, the advantages of dealing with stationary random functions should be opted for, based on the canonic expansion (3.1):

The random functions representing (scalar or vectorial) accelerograms can be fully characterized in this case by their auto-correlation (or covariance) functions or by their Fourier transforms, the terms of the ("power") spectrum density matrix. One can consider in this connection the three variants referred to in subsection 3.1. Given the fact that the use of stationary random functions like $\boldsymbol{w}_g{}^{(s)}{}_k(t)$ is well suited for the generation of artificial accelerograms, the canonic expansion (3.1), in connection with the classical Wiener – Khinchin relations, will be considered as a basis further on.

In case of using scalar stationary random functions for the representation of (single degree of freedom) seismic ground motions, a couple of frequently used expressions of the auto-correlation function $b[w_g{}^{(s)}{}_k(t);\ t_n]$ and of the corresponding spectrum density function $s[w_g{}^{(s)}{}_k(t);\ \omega_m]$ is represented by the well known Kanai – Tajimi expressions

$$b[w_g{}^{(s)}{}_k(t);\ t_n,\ a_k,\ \alpha_k,\ \beta_k] = a_k{}^2 \times \exp\ (-\alpha_k\ |t_n|)\ [\cos\ (\beta_k\ |t_n|) + (\alpha_k/\beta_k)\ \sin\ (\beta_k\ |t_n|)] \quad (4.1)$$

$$s[w_g{}^{(s)}{}_k(t);\ \omega_m,\ a_k,\ \alpha_k,\ \beta_k] = (2\ a_k{}^2\ /\ \pi)\ (\alpha_k{}^2 + \beta_k{}^2)\ /\ \{\omega_m{}^4 + 2\ (\alpha_k{}^2 - \beta_k{}^2)\ \omega_m{}^2 + (\alpha_k{}^2 + \beta_k{}^2)^2\} \quad (4.2)$$

where a_k represents the r.m.s. value of $w_g{}^{(s)}{}_k(t)$. It may be recalled here that the Kanai – Tajimi expressions have an attractive physical sense: they may correspond to the case when a white – noise function of time, representing the base motion, is filtered by a visco-elastic (Kelvin – Voigt type), inverted pendulum type, SDOF linear dynamic system, having an undamped circular frequency equal to $(\alpha_k{}^2 + \beta_k{}^2)^{1/2}$ and a fraction of critical damping equal to $\alpha_k\ /\ (\alpha_k{}^2 + \beta_k{}^2)^{1/2}$.

Note that the subscript $_k$ of the expansion (3.1) and the superscript $^{(s)}$ of the same will be dropped further on, since following developments are related to a single (arbitrary) term of that expansion. Note also that the variable ω_m of relations (3.3), (3.4) is replaced subsequently by the shorter ω.

4.2. Basic prerequisites

The goal of subsequent developments is to propose a stochastic model of seismic motions of the ground, considered in its turn as a continuum in which wave propagation produces at different points motions that are non – synchronous. A summary view of developments in this field was presented e.g. in (Zerva & al., 1994), where cross-correlations between motions along parallel directions at different points were mainly dealt with. Another example of developments of the same kind is presented in (Spudich, 1994). As other examples, directly related to the developments of this paper, one can consider the early developments of

- (Bălan & al. 1977), where a stochastic model of non – synchronous motion at the various support points of a multi – span bridge was at the origin of generation of non – synchronous artificial accelerograms at these points;
- (Pârvu & al. 1978), where a stochastic model of non – synchronous motion at the various support points of a multi – span bridge was at the origin of a parametric analysis of the modal participation factors defining the seismic input.

It may be mentioned in this connection that the developments referred to considered motions along (single) parallel directions at different points, located along a same alignment. The following developments, which update those of (Sandi, 1982) and (Sandi, 2005), are devoted to proposing a more complete model, in a frame where all six relevant degrees of freedom of ground motion are dealt with, for points located in a 3D continuum. They represented the basis for drafting the Annex C of the Romanian Code (MLPAT, 1992), where the characteristics of seismic input were specified with explicit consideration of the simultaneous motion along three degrees of freedom that are characteristic for a foundation mat: translation along two orthogonal horizontal directions and rotation in the horizontal plane. This approach made it possible among other to develop a critical look at the empirical tradition of adopting a conventional eccentricity for the conventional seismic forces.

A basic step in order to build the desired stochastic model is represented by the postulation of the cross-correlation or (which is equivalent) of the cross – spectrum density matrix for the ground motion components at two different points. Given the engineering requirements, one should consider at the same time translation and rotation components of the local motion at various points of the ground. One will consider here two orthogonal horizontal axes *Ox* and *Oy* and a vertical axis *Oz*, along which the translation displacements are *u*, *v*, and *w*, while the rotation displacements are φ and χ for two orthogonal horizontal infinitesimal segments and the average rotation in the horizontal plane, ψ, respectively, defined by the relations

$$\varphi = \partial w / \partial x \quad (4.3a)$$

$$\chi = \partial w / \partial y \quad (4.3b)$$

$$\psi = (\partial v / \partial x - \partial u / \partial y) / 2 \quad (4.3c)$$

In order to develop a (stationary) stochastic model of ground motion, corresponding to a continuum model of ground, it is necessary to correspondingly develop expressions of the cross - correlations (or of their Fourier transforms) between the various components of the vector $\boldsymbol{u_g}(P, t)$ of displacements or of the vector $\boldsymbol{w_g}(P, t) = \boldsymbol{\ddot{u}_g}(P, t)$ of accelerations (whereby *P* means a condensed representation of the spatial coordinates of a point).

The (transpose of the) displacement vector $\boldsymbol{u_g}(P, t)$ consists of following components:

$$\boldsymbol{u_g}^T(P, t) = [u(P, t), v(P, t), w(P, t), \chi(P, t), -\varphi(P, t), \psi(P, t)] \quad (4.4)$$

(the superscript ^{T} means, again, the transpose). The way to do this is to postulate cross - correlation characteristics for the translation components $u(P, t)$, $v(P, t)$, $w(P, t)$ and to derive, on the basis of the relations (4.3), similar relations for the matrix components where rotation components $\varphi(P, t)$, $\chi(P, t)$, $\psi(P, t)$ are also involved.

The classical cross spectrum density matrix of ground acceleration $\boldsymbol{S}[\boldsymbol{w}_g(t); P_1, P_2, \omega_m]$, related to a couple of points P_1 and P_2, will concern the dyadic product $\boldsymbol{w}_g(P_1, t) \times \boldsymbol{w}_g^T(P_2, t)$, whereby the acceleration vector $\boldsymbol{w}_g(P, t) = \boldsymbol{\ddot{u}_g}(P, t)$, corresponds to the displacement vector $\boldsymbol{u_g}(P, t)$ defined by the expression (4.4).

4.3. Stochastic ground motion model proposed

Following basic properties of the vector $\boldsymbol{w_g}(P, t)$ will be admitted in this connection:

a) stationarity with respect to time;
b) plane, horizontal, free boundary of the ground (the ground model concerns a half-space consisting of a sequence of plane, parallel, horizontal, layers);
c) homogeneity of properties with respect to the coordinates x, y;
d) isotropy with respect to the horizontal directions x, y.

Developing of the cross spectrum density matrix is to proceed as follows (Sandi, 1982):

a) postulating expressions for the first three (translation) components of the ground displacement vector $\boldsymbol{u}_g^T(P, t)$ (4.4) or of the corresponding ground acceleration vector $\boldsymbol{w}_g^T(P, t)$;
b) deriving, on this basis and on the basis of expressions (4.3), of similar expressions for the matrix components corresponding also (partially or exclusively) to the last three (rotation) components of the vector $\boldsymbol{u}_g^T(P, t)$ (4.4).

Following assumptions were adopted in order to solve the step (a):

1. There is no cross-correlation between motion components along the orthogonal directions x, y, z, at any points.
2. Two basic similitude criteria concerning the wave propagation phenomenon are to be considered (Sandi, 1975):
   - the phase lag criterion $s_{\Delta\varphi}$ and

$$s_{\Delta\varphi} = (c\ \Delta\varphi) / (\omega\, d) \tag{4.5}$$

   - the local rotation amplitude criterion s_θ,

$$s_\theta = (c\ \theta_0) / (\omega\, u_0) \tag{4.6}$$

where following notations were used:

- c: a wave propagation velocity;
- d: distance between points;
- ω: circular frequency;
- $\Delta\varphi$: phase lag;
- u_0: amplitude of a translation component;
- θ_0: amplitude of a rotation component.

3. Since the ratio d / c which occurs in the expression is equivalent to a propagation time denoted T^*, it is interesting to redefine this parameter specifying in more detail the direction along which the distance d, versus the direction of motion, are considered; a solution in this respect is given by the introduction of an equivalent propagation time T^*, given by the expression

$$T^*(c_l,\ c_t,\ d_l,\ d_t,\ z_0) = [<d_l\ /\ c_l(z_0)>^2 + <d_t\ /\ c_t(z_0)>^2]^{1/2} \tag{4.7}$$

where following notations were used:

- c_l: equivalent propagation velocity, homologous to that of P-waves;
- c_t: equivalent propagation velocity, homologous to that of S-waves;
- d_l: projection of distance between both points along the direction of motion considered;
- d_t: projection of distance between both points across the direction of motion considered.

4. Based on expressions (4.5) and (4.7), the basic characteristic of cross correlation between the (translation) accelerations at different points, represented by a coherence factor ρ, dealt with subsequently in more detail, will depend on a phase lag $\Delta\varphi$ given by the expression

$$\Delta\varphi = s_{\Delta\varphi}\ \omega\ T^* \tag{4.8}$$

where $s_{\Delta\varphi}$ is a non-dimensional similitude criterion, as defined by the expression (4.5), having a value that is specific to the case dealt with.

The cross - coherence of motion along parallel (translation) directions at different points is now to be postulated, and this is a difficult point, given the complexity of the wave propagation phenomenon across the various sequences of geological layers. A simple, strongly idealized, solution is proposed in this connection, assuming a similarity between this latter characteristic and the auto - correlation expression (4.1), in view of the features of the wave propagation phenomenon. Given the similitude criteria of wave propagation, the phase lag criterion $s_{\Delta\varphi}$ (4.5) and its consequence (4.8) are considered for the basic argument in this connection.

5. Based on previous developments and considerations, assuming due to the features of the wave propagation phenomenon, a kind of similitude between the auto-correlation function $b[w_g{}^{(s)}{}_k(t);\ t_n,\ a_k,\ \alpha_k,\ \beta_k]$ (4.1) and the expression of the coherence factor for two points located at a same depth z_0, an expression

$$\rho\ (\upsilon,\ \omega,\ T^*) = \exp\ (-\ \upsilon\ T^*)\ \{\cos\ (\omega\ T^*) + (\upsilon/\omega)\sin\ (\omega\ T^*)\} \tag{4.9}$$

is proposed, where the newly introduced parameter υ (which is homologous to α of expression (4.1)), could be a function $\upsilon(\omega, z_0)$, or might be a constant, or a function of ω only. A first proposal in this view is to assume a value

$$\upsilon \approx 0.5\ \omega. \tag{4.10}$$

6. Adopting the indices 1 through 6 for the sequence of components of expression (4.4), the terms of the cross spectrum density matrix corresponding to the indices 1 through 3 will be

$$s_{11}[\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = s_{11}[\boldsymbol{w}_g;\ \omega,\ 0,\ 0,\ z_0]\ \rho\{\upsilon,\ \omega,\ T^*(c_l,\ c_t,\ \Delta x,\ \Delta y)\} \tag{4.11a}$$

$$s_{22}[\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = s_{22}[\boldsymbol{w}_g;\ \omega,\ 0,\ 0,\ z_0]\ \rho\{\upsilon,\ \omega,\ T^*(c_t,\ c_l,\ \Delta x,\ \Delta y)\} \tag{4.11b}$$

$$s_{33}[\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = s_{33}[\boldsymbol{w}_g;\ \omega,\ 0,\ 0,\ z_0]\ \rho\{\upsilon,\ \omega,\ T^*(c_z,\ c_z,\ \Delta x,\ \Delta y)\} \tag{4.11c}$$

$$s_{ij}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = 0 \quad (\text{for } i, j = 1, 2, 3,\ i \neq j) \tag{4.11d}$$

where c_z means an equivalent propagation velocity for oscillations in the vertical direction.

7. Due to the postulation of isotropy, the conditions and expressions

$$s_{11}[\boldsymbol{w}_g;\ \omega,\ 0,\ 0,\ z_0] = s_{22}[\boldsymbol{w}_g;\ \omega,\ 0,\ 0,\ z_0] = s_h[\boldsymbol{w}_g;\ \omega,\ z_0] \tag{4.12a}$$

$$s_{33}[\boldsymbol{w}_g;\ \omega,\ 0,\ 0,\ z_0] = s_v[\boldsymbol{w}_g;\ \omega,\ z_0] \tag{4.12b}$$

are considered further on ($s_h[\boldsymbol{w}_g;\ \omega,\ z_0]$: spectrum density for horizontal translation acceleration; $s_v[\boldsymbol{w}_g;\ \omega,\ z_0]$: spectrum density for vertical translation acceleration).

8. The expressions concerning also, or only, rotation components (indices 4 through 6 for components of (4.4)) will be (skipping arguments where acceptable)

$$s_{14} = s_{15} = s_{24} = s_{25} = s_{36} = s_{41} = s_{42} = s_{46} = s_{51} = s_{52} = s_{56} = s_{63} = s_{64} = s_{65} = 0 \tag{4.13a}$$

$$s_{16}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = -\ s_{61}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = -\ (1/2)\ s_h[\boldsymbol{w}_g;\ \omega,\ z_0]\ (\partial/\partial\Delta y)\ \rho\{\upsilon,\ \omega,\ T^*(c_l,\ c_t,\ \Delta x,\ \Delta y)\} \tag{4.13b}$$

$$s_{26}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = -\ s_{62}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = (1/2)\ s_h[\boldsymbol{w}_g;\ \omega,\ z_0]\ (\partial/\partial\Delta x)\ \rho\{\upsilon,\ \omega,\ T^*(c_t,\ c_l,\ \Delta x,\ \Delta y)\} \tag{4.13c}$$

$$s_{34}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = -\ s_{43}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = s_v[\boldsymbol{w}_g;\ \omega,\ z_0]\ (\partial/\partial\Delta y)\ \rho\{\upsilon,\ \omega,\ T^*(c_z,\ c_z,\ \Delta x,\ \Delta y)\} \tag{4.13d}$$

$$s_{35}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = -\ s_{53}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = -\ s_v[\boldsymbol{w}_g;\ \omega,\ z_0]\ (\partial/\partial\Delta x)\ \rho\{\upsilon,\ \omega,\ T^*(c_z,\ c_z,\ \Delta x,\ \Delta y)\} \tag{4.13e}$$

$$s_{44}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = -\ s_v[\boldsymbol{w}_g;\ \omega,\ z_0]\ (\partial^2/\partial\Delta y^2)\ \rho\{\upsilon,\ \omega,\ T^*(c_z,\ c_z,\ \Delta x,\ \Delta y)\} \tag{4.13f}$$

$$s_{55}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = -\ s_v[\boldsymbol{w}_g;\ \omega,\ z_0]\ (\partial^2/\partial\Delta x^2)\ \rho\{\upsilon,\ \omega,\ T^*(c_z,\ c_z,\ \Delta x,\ \Delta y)\} \tag{4.13g}$$

$$s_{66}\ [\boldsymbol{w}_g;\ \omega,\ \Delta x,\ \Delta y,\ z_0] = -\ (1/4)\ s_h[\boldsymbol{w}_g;\ \omega,\ z_0] < (\partial^2/\partial\Delta x^2)\ \rho\{\upsilon,\ \omega,\ T^*(c_t,\ c_l,\ \Delta x,\ \Delta y)\} + (\partial^2/\partial\Delta y^2)\ \rho\{\upsilon,\ \omega,\ T^*(c_l,\ c_t,\ \Delta x,\ \Delta y)\} > \tag{4.13h}$$

It may be noted here that the partial derivatives introduced have following expressions:

$(\partial/\partial\Delta x)\ \rho\{\upsilon,\ \omega,\ T^*(c_t,\ c_l,\ \Delta x,\ \Delta y)\} = -\{(\upsilon^2 + \omega^2)/\omega)\} \times \exp\{-\upsilon\, T^*(c_t, c_l, \Delta x, \Delta y)\} \sin\{\omega\, T^*(c_t,\ c_l,\ \Delta x,\ \Delta y)\} \times \{(\partial/\partial\Delta x)\ T^*(c_t,\ c_l,\ \Delta x,\ \Delta y)\}$ (4.14a)

$(\partial/\partial\Delta y)\ \rho\{\upsilon,\ \omega,\ T^*(c_l,\ c_t,\ \Delta x,\ \Delta y)\} = -\{(\upsilon^2 + \omega^2)/\omega)\} \times \exp\{-\upsilon\, T^*(c_l, c_t, \Delta x, \Delta y)\} \sin\{\omega\, T^*(c_l,\ c_t,\ \Delta x,\ \Delta y)\} \times \{(\partial/\partial\Delta y)\ T^*(c_l,\ c_t,\ \Delta x,\ \Delta y)\}$ (4.14b)

$(\partial^2/\partial\Delta x^2)\ \rho\{\upsilon,\ \omega,\ T^*(c_t,\ c_l,\ \Delta x,\ \Delta y)\} = -\{(\upsilon^2 + \omega^2)/\omega)\}\ \exp\{-\upsilon\, T^*(c_t, c_l, \Delta x, \Delta y)\} \times \subset<\omega \cos\{\omega\, T^*(c_t, c_l, \Delta x, \Delta y)\} - \upsilon \sin\{\omega\, T^*(c_t, c_l, \Delta x, \Delta y)\}> \times \{(\partial/\partial\Delta x)\ T^*(c_x,\ c_y,\ \Delta x,\ \Delta y)\}^2 + \{\sin\{\omega, T^*(c_x, c_y, \Delta x, \Delta y)\}\ (\partial^2/\partial\Delta x^2)\ T^*(c_x, c_y, \Delta x, \Delta y)\}\supset$ (4.14c)

$\partial^2/(\partial\Delta x\partial\Delta y)\ \rho\{\upsilon,\ \omega,\ T^*(c_t,\ c_l,\ \Delta x,\ \Delta y)\} = -\{(\upsilon^2 + \omega^2)/\omega)\}\ \exp\{-\upsilon\, T^*(c_x, c_y, \Delta x, \Delta y)\} \times \subset<\omega \cos\{\omega\, T^*(c_x,\ c_y,\ \Delta x,\ \Delta y)\} - \upsilon \sin\{\omega\, T^*(c_x, c_y, \Delta x, \Delta y)\}> \times <\{(\partial/\partial\Delta x)\ T^*(c_x, c_y, \Delta x, \Delta y)\} \times \{(\partial/\partial\Delta y)\ T^*(c_x, c_y, \Delta x, \Delta y)\}> + \{\sin\{\omega\, T^*(c_x,\ c_y,\ \Delta x,\ \Delta y)\}\ (\partial^2/\partial\Delta x\partial\Delta y)\ T^*(c_x,\ c_y,\ \Delta x,\ \Delta y)\}\supset$ (4.14d)

$(\partial^2/\partial\Delta y^2)\ \rho\{\upsilon,\ \omega,\ T^*(c_x,\ c_y,\ \Delta x,\ \Delta y)\} = -\{(\upsilon^2 + \omega^2)/\omega)\}\ \exp\{-\upsilon\, T^*(c_x, c_y, \Delta x, \Delta y)\} \times \subset<\omega \cos\{\omega\, T^*(c_x, c_y, \Delta x, \Delta y)\} - \upsilon \sin\{\omega\, T^*(c_x, c_y, \Delta x, \Delta y)\}> \times \{(\partial/\partial\Delta y)\ T^*(c_x,\ c_y,\ \Delta x,\ \Delta y)\}^2 + \{\sin\{\omega, T^*(c_x, c_y, \Delta x, \Delta y)\}\ (\partial^2/\partial\Delta y^2)\ T^*(c_x, c_y, \Delta x, \Delta y)\}\supset$ (4.14e)

(where c_x, c_y may play the role of c_l, c_t, c_z, as suited)

$(\partial/\partial\Delta x)\ T^*(c_x, c_y, \Delta x, \Delta y) = (\Delta x / c_x^2) / (\Delta x^2 / c_x^2 + \Delta y^2 / c_y^2)^{1/2}$ (4.14f)

$(\partial/\partial\Delta y)\ T^*(c_x, c_y, \Delta x, \Delta y) = (\Delta y / c_y^2) / (\Delta x^2 / c_x^2 + \Delta y^2 / c_y^2)^{1/2}$ (4.14g)

$\partial^2/\partial\Delta x^2)\ T^*(c_x, c_y, \Delta x, \Delta y) = (\Delta y^2 / c_x^2 c_y^2) / (\Delta x^2 / c_x^2 + \Delta y^2 / c_y^2)^{3/2}$ (4.14h)

$\partial^2/(\partial\Delta x\partial\Delta y)\ T^*(c_x, c_y, \Delta x, \Delta y) = -(\Delta x\Delta y / c_x^2 c_y^2) / (\Delta x^2 / c_x^2 + \Delta y^2 / c_y^2)^{3/2}$ (4.14i)

$(\partial^2/\partial\Delta y^2)\ T^*(c_x, c_y, \Delta x, \Delta y) = (\Delta x^2 / c_x^2 c_y^2) / (\Delta x^2 / c_x^2 + \Delta y^2 / c_y^2)^{3/2}$ (4.14j)

4.4. Some illustrative applications

4.4.1. Applications to analysis exercises

Two applications of the model in its early stages of development, (Bălan & al. 1977) and (Pârvu & al. 1978) respectively, are mentioned at this place. Both refer to non-synchronous seismic input to multi-span bridges.

The application presented in (Bălan & al. 1977) was concerned with the generation of non-synchronous accelerograms for the model of a multi-span bridge. The accelerograms at the ground – structure contact areas relied on a stochastic model according to which the simultaneous motions at different contact areas were defined. The accelerograms generated were characterized by a stronger correlation of lower frequency spectral components and by a weaker correlation of higher freqency components.

The application presented in (Pârvu & al. 1978) was concerned with the analysis of input and response in case of the trans - Danube bridge of Fetești, Romania, designed in 1977. The primary goal of analysis was to define the modal input for the natural modes of the structure, keeping in view the coherence characteristics corresponding to the system of ground - structure contact areas. Since there were no data available on the parameter *c* (wave propagation velocity) of relation (4.5) and on its correspondents of further equations, a parametric approach was adopted, which resulted in the computation of the modal participation factors as functions of the equivalent propagation velocity.

4.4.2. Application to code development

A different application is related to the development of the Annex C of the Romanian earthquake resistant design code (MLPAT 1992). This annex is concerned with the spatial analysis of structural performance under earthquake loading, keeping explicitly into account the simultaneous action along the different degrees of freedom of the ground -

structure interface. A reference scheme is given in Figure 1. The basic structural model adopted relies on the assumption of a 3DOF ground – structure rigid interface motion, having two translation components, along the axes x and y, and one rotation component, around the axis θ.

A basic relation, specifying the design accelerations, is

$$w_{kr} = c_{kr}\, \mathrm{g} \tag{4.15}$$

where following symbols appear:

- w_{kr}: conventional design accelerations along the degrees of freedom of index k, corresponding to the natural modes of index r;
- c_{kr}: corresponding (non – dimensional) seismic factors;
- g: acceleration of gravity.

The seismic factors c_{kr} are given by the expression

$$c_{kr} = k_s\, \beta_r\, \psi\, \eta'_{kr} \tag{4.16}$$

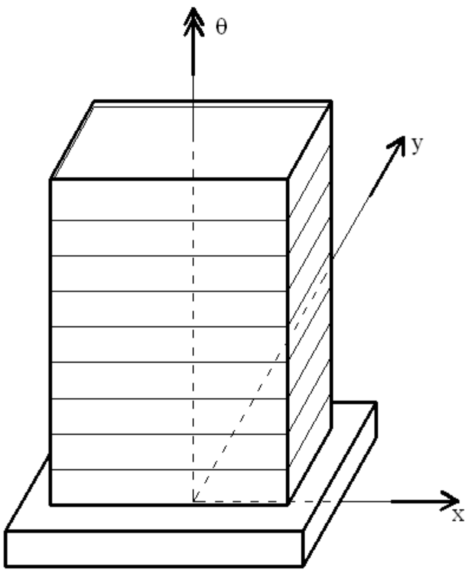

Fig. 1. Scheme of degrees of freedom considered for spatial analysis

where following new symbols appear:

- k_s: basic seismic factor (fraction of gravity acceleration);
- β_r: dynamic factor corresponding to mode of index r;
- ψ: reduction factor accounting for inelastic performance;
- η'_{kr}: shape factor.

The shape factors are given by an expression

$$\eta'_{kr} = v_{kr}\, p_r \tag{4.17}$$

where following new symbols appear:

- v_{kr}: components of eigenvectors along the degrees of freedom of index k, corresponding to the natural modes of index r;
- p_r: modal participation factors.

The modal participation factors are given by an expression

$$p_r = (p_{rx}^2 + p_{ry}^2 + p_{r\theta}^2)^{1/2} \tag{4.18}$$

where

$$p_{rx} = (\Sigma_k\, m_k\, v_{kr}\, \gamma_{kx}) / A_r \tag{4.19a}$$

$$p_{ry} = (\Sigma_k\, m_k\, v_{kr}\, \gamma_{ky}) / A_r \tag{4.19b}$$

$$p_{r\theta} = [\pi\, 2^{1/2}\, \xi_r / (c_e\, T_r)]\ \{\Sigma_k\, [m_k\, (x_k\, \gamma_{ky} - y_k\, \gamma_{kx})\,]\} / A_r \tag{4.19c}$$

where following new symbols appear:

- m_k: mass associated to the DOF k;
- γ_{kx}, γ_{ky}, $\gamma_{k\theta}$: cosines of the degrees of freedom k to the three axes;
- ξ_r: factor referred to subsequently, plotted in Figure 2;
- c_e: equivalent wave propagation velocity;
- T_r: eigenperiod for the r-th mode;
- A_r: modal norm, given by the expression

$$A_r i = \ \Sigma_k\, m_k\, v_{kr}^2 \tag{4.20}$$

The document referred to presents also more general expressions, for cases when the inertia matrix is non – diagonal, or when mechanical moments of inertia are associated to the degrees of freedom. An expression of ξ_r is given too. In the absence of special studies, following values are recommended for c_e: 200 m/s for soft soils, 400 m/s for medium soils and 600 m/s for hard soils. It is recommended to use the expression (4.19c) only in cases when $L_c /(c_e\ T_r) < 1 / 3$. Else, special studies are recommended.

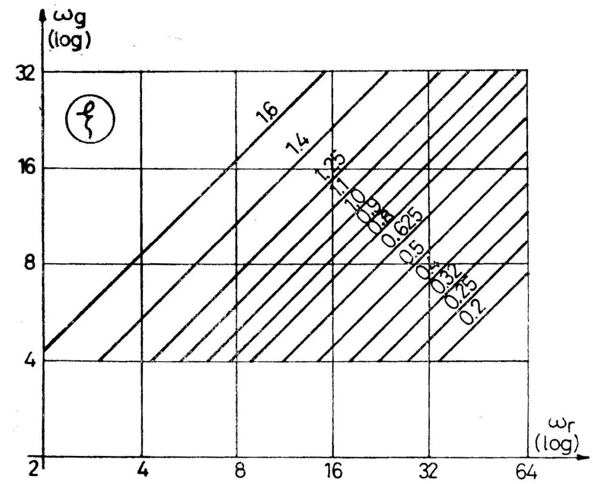

Fig. 2. Diagram of parameter ξ (ωr: natural circular frequency of r-th mode; ωg: dominant circular frequency of ground motion)

4.4.3. Application to the case of a large span structure

The analysis of random (stationary) oscillation of a large span structure is discussed for this application. The case dealt with can no longer be analyzed on the basis of relations (4.15) to (4.20), because the kinematic assumption of a rigid body motion of the ground – structure interface becomes unrealistic. The main aspect dealt with is that, of a connection between the characterization of the stochastic model presented in Subsection 4.3 and the specification of the input to the equations of motion of a special civil engineeering structure.

The structure dealt with belongs to the main hall of the ROMEXPO (earlier EREN) exhibition campus of Bucharest. The hall is covered by a 94.2 m span circular dome steel structure, supported at its turn by a system of 32 (internal) main equi-distant columns. A vertical and a horizontal plane sections are presented in Figure 3. The external columns play a secondary supporting role. Further information on the dome supporting structure is provided in Figure 4.

The structure was repeatedly affected by the successive strong Vrancea earthquakes of 1977.03.04, 1986.08.30, 1990.05.30 and 1990.05.31. Due to its importance, among other, its dynamic characteristics started to be monitored already in the pre-earthquake stage, and this monitoring was continued after the earthquakes as well as after performing strengthening interventions.

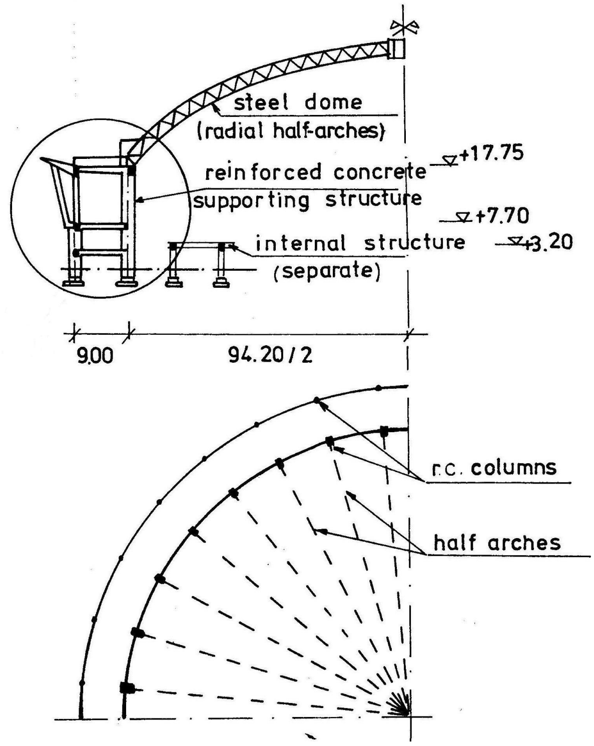

Fig. 3. Vertical and horizontal sections for the structure of the EREN / ROMEXPO main exhibition hall (for details on circle zone, see Figure 4)

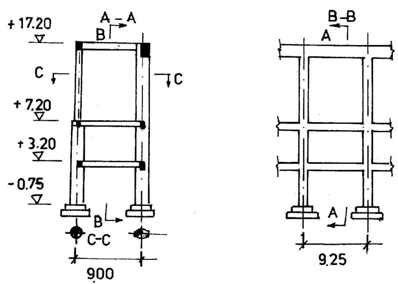

Fig. 4. Details on circle zone of Figure 3. A – A: in the radial plane; B – B: in the tangential plane

The evolution of the dynamic characteristics (before strong earthquakes, after earthquakes and after strengthening actions) was first presented in (Sandi & al.

1986). Additional data, based on updating after the 1990 strong earthquakes, were presented in (Sandi & al. 2002). A summary view of the evolution of natural periods is reproduced from the latter source in Table 1. The data presented put to evidence the initial axial symmetry, the flexibilization due to earthquakes, the loss of axial symmetry after the 1977.03.04 event, the effects of the two strengthening interventions, as well as the effects of the events of 1986 and 1990.

The almost perfect initial axial dynamic symmetry of the structure leads to the orthogonality of four main categories of oscillations (pertaining to corresponding subspaces of the space of posssible deformations of the structure, S_s) , which can be characterized essentially as in Table 2. *Note*: in case a perfect axial symmetry is assumed, further subspaces of orthogonal categories of deformations and oscillations (e.g. higher order kinds of ovalization corresponding to a Fourier expansion) can be identified, but those will be of secondary, if not even negligible, importance for the dynamic structural performance.

Table 1. *Predominant oscillation periods (s), determined under various situations for the structure of the main "EREN / ROMEXPO" exhibition hall*

Oscillation direction (DOF)	Recording moment					
	Before 1977.03.04 'quake (July '76)	After 1977.03.04 'quake (Mar. '77)	After provisional strength'ng (steel bracing) (April '77)	After final strength'ng (r.c. spatial frame) (July '84)	After 1986.08.30 'quake (Sept. '86)	After 1990.05 'quakes (July '93)
N - S ring translation	.60	1.08	.78	.55	.65	.66
E - W ring translation	.60	.98	.74	.52	.65	.72
In plane ring rot'n	.41	.94	.59	.43	.52	.52
Ring ovalization	.35	.36	.36	.34	.39	.41

Table 2. Subspaces of structural deformation for which the oscillations of the EREN / ROMEXPO structure can be analyzed independently

Type of deformation	Reference axis name	Subspace name
horizontal translation along the W - E direction	*Ox*	S_x
horizontal translation along the S - N direction	*Oy*	S_y
rotation with respect to the vertical axis	*Oz*	S_θ
second order ovalization (in the horizontal plane) of the main dome supporting ring	-	S_v

The examination of the records of ambient vibrations (which had almost purely sinusoidal shapes) made it possible to state that, for practical engineering analyses, it is sufficient to consider just the oscillations corresponding to the fundamental modes pertaining to each of the four subspaces referred to before. An argument in favour of this statement is provided also by the outcome of an illustrative analysis of non – linear seismic oscillations for a fragment of B – B type of Figure 4 of the supporting structure. This was subjected to an artificial accelerogram having a dominant frequency of 2 Hz and a peak ground acceleration of 2 m/s^2. The acceleration time history (Figure 5) and the displacement time history (Figure 6) at the main ring level are almost sinusoidal.

An appropriate way to analyze the stochastic dynamic performance of the structure on the basis of the developments of Subsection 4.3 is to express the basic equations in terms of corresponding Fourier transforms and to specify the input data in appropriate terms. Some reference equations of this kind, used subsequently, rely on the developments of (Sandi, 1983).

The solution of the system of linear equations of motion of a structure idealized as a dynamic system with a finite number, n, of degrees of freedom, is expressed on the basis

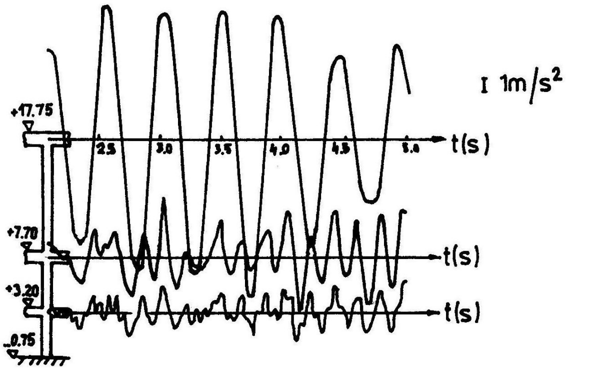

Fig. 5. Time history of accelerations for structure of Figure 4 (B – B) under artificial ground motion

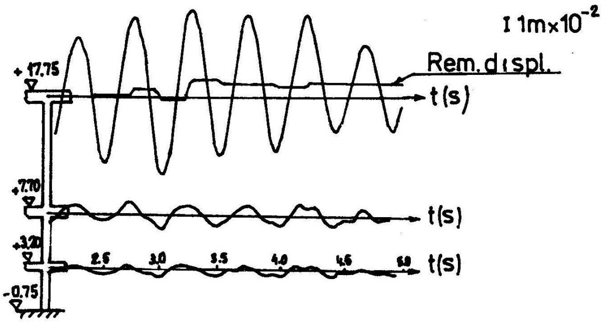

Fig. 6. Time history of displacements for structure of Figure 4 (B – B) under artificial ground motion

of the solutions of the corresponding eigenvalue problem, as a linear combination

$$\boldsymbol{u}^{\sim}(\omega) = \Sigma_r^{1,n}\, \boldsymbol{v}_r\, q_r^{\sim}(\omega) \qquad (4.21)$$

where following symbols appear:

- $\boldsymbol{u}^{\sim}(\omega)$: Fourier transform of the displacement vector $\boldsymbol{u}(t)$;
- ω: circular frequency;
- $\boldsymbol{v}_r$: normalized eigenvector of r-th order (eigenvectors are assumed to be solutions of a classical linear eigenvalue problem, ergo to be real and constant, and to be normalized with respect to the mass matrix $\boldsymbol{M}$);
- $q_r^{\sim}(\omega)$: Fourier transforms of the corresponding normal coordinates $q_r(t)$.

In case of such a structure, if it is subjected to the action of ground motion, represented by a vector $\boldsymbol{u}_g(t)$, the solution of the system of equations of motion, expressed in terms of absolute displacements $\boldsymbol{u}_a(t)$, is

$$\boldsymbol{u}_a^{\sim}(\omega) = \Sigma_r^{1,n}\, \{\boldsymbol{v}_r \boldsymbol{v}_r^T / [\, \lambda_r^{\sim}(\omega) - \omega^2]\}\, \boldsymbol{K}_g^{\sim}(\omega)\, \boldsymbol{u}_g^{\sim}(\omega) \qquad (4.22)$$

where following new symbols appear:

- $\boldsymbol{u}_a^{\sim}(\omega)$: Fourier transform of the absolute displacement vector $\boldsymbol{u}_a(t)$, pertaining to the space of degrees of freedom of a structure in elevation, S_s;
- $\boldsymbol{u}_g^{\sim}(\omega)$: Fourier transform of the displacement vector $\boldsymbol{u}_g(t)$ pertaining to the space of degrees of freedom of the ground – structure interface, S_g;
- $\lambda_r^{\sim}(\omega)$: Fourier transform of the eigenvalue of r-th order (these functions are in the general case complex and have, for anelastic, exothermal, structure materials, positive values of their real and imaginary parts);
- $\boldsymbol{K}_g^{\sim}(\omega)$: Fourier transform of the cross (usually rectangular) stiffness matrix of the structure.

Note:

- the spaces S_s and S_g have no intersection;
- a column of the matrix $\boldsymbol{K}_g^{\sim}(\omega)$ represents the Fourier transform of the system of forces applied along the degrees of freedom corresponding to the space S_s, in case a unit (amplitude) displacement is imposed along the corresponding degree of freedom of the space S_g;
- conversely, a row of the matrix $\boldsymbol{K}_g^{\sim}(\omega)$ represents the Fourier transform of the system of forces

applied along the degrees of freedom corresponding to the space S_g, in case a unit (amplitude) displacement is imposed along the corresponding degree of freedom of the space S_s.

The equation (4.22) is valid also for the homologous acccelerations, as

$$\boldsymbol{w}_a^{\sim}(\omega) = \Sigma_r^{1,n} \{\boldsymbol{v}_r \boldsymbol{v}_r^T / [\lambda_r^{\sim}(\omega) - \omega^2]\} \boldsymbol{K}_g^{\sim}(\omega) \boldsymbol{w}_g^{\sim}(\omega) \tag{4.23}$$

where the Fourier transforms of the second order derivatives

$$\boldsymbol{w}_a(t) = \ddot{\boldsymbol{u}}_a(t) \tag{4.24a}$$

$$\boldsymbol{w}_g(t) = \ddot{\boldsymbol{u}}_g(t) \tag{4.24b}$$

appear.

In case one accepts the pragmatic assumption according to which the subspaces S_x (rigid translation of main ring along the axis Ox), S_y (the same for Oy), S_θ (rotation of main ring with respect to the central vertical axis) and S_v (second order ovalization of the main ring in the horizontal plane) are all mono-dimensional,

- the normalized eigenvectors $\boldsymbol{v}_r$ will reduce to scalars $1/m_a^{1/2}$, which will take values $1/m_{ax}^{1/2}$ (m_{ax}: overall mass), $1/m_{ay}^{1/2}$ (m_{sy}: overall mass), $1/m_{a\theta}^{1/2}$ ($m_{sa\theta}$: overall mechanical moment of inertia), and $1/m_{av}^{1/2}$ (m_{av}: equivalent mass, determined by means of an integral corresponding to the specific expression of the kinetic energy), respectively;
- the dyadic products $\boldsymbol{v}_r\, \boldsymbol{v}_r^T$ wil be replaced by scalars $1/m_a$ ($1/m_{ax}$, $1/m_{ay}$, $1/m_{a\theta}$, and $1/m_{av}$ respectively);
- the Fourier transforms of the accceleration vectors $\boldsymbol{w}_a^{\sim}(\omega)$ will reduce to the scalar functions $w_{ax}^{\sim}(\omega)$, $w_{ay}^{\sim}(\omega)$, $w_{a\theta}^{\sim}(\omega)$ and $w_{av}^{\sim}(\omega)$, while
- the corrresponding matrices $\boldsymbol{K}_g^{\sim}(\omega)$ will reduce to row vectors $\boldsymbol{k}_g^{\sim}(\omega)$, which can be denoted $\boldsymbol{k}_{gx}^{\sim}(\omega)$, $\boldsymbol{k}_{gy}^{\sim}(\omega)$, $\boldsymbol{k}_{g\theta}^{\sim}(\omega)$, and $\boldsymbol{k}_{gv}^{\sim}(\omega)$ respectively.

For each of the subspaces referred to, the equation (4.23) will become

$$w_a^{\sim}(\omega) = \{(1/m_a) / [\lambda^{\sim}(\omega) - \omega^2]\} \boldsymbol{k}_g^{\sim}(\omega) \boldsymbol{w}_g^{\sim}(\omega) \tag{4.23'}$$

where $\lambda^{\sim}(\omega)$ is the Fourier transform for the unique eigenvalue corresponding to that subspace, while the vector $\boldsymbol{w}_g^{\sim}(\omega)$ is the same for all subspaces.

The degrees of freedom of the space S_g selected for engineering analyses of this structure would correspond to translation components along the axes Ox and Oy, for each of the 32 contact areas between ground and main columns (a total of 64 DOF's). Other DOF's would be much less relevant. The [64 × 64] spectral density matrix $\boldsymbol{S}_g^{\sim}(\omega)$, corresponding to a stochastic characterization for stationary random ground motion, will consist of terms $s_{11}[\boldsymbol{w}_g;\, \omega,\, \Delta x,\, \Delta y,\, z_0]$ (4.11a) and $s_{22}[\boldsymbol{w}_g;\, \omega,\, \Delta x,\, \Delta y,\, z_0]$ (4.11b), while $s_{12}[\boldsymbol{w}_g;\, \omega,\, \Delta x,\, \Delta y,\, z_0] = 0$ (4.11d), computed for the values of distances Δx and Δy corresponding to the various couples of ground – column contact areas.

The result of computations will be expressed by the spectrum densities $s\,[w_a;\, \omega]$, computed for each of the subspaces S_x, S_y, S_θ and S_v, on the basis of the equation (Pugachov 1960), (Sandi 1983)

$$s\,[w_a;\, \omega] = \{(1/m_a^2) / |\, [\lambda^{\sim}(\omega) - \omega^2]\, |^2\} \boldsymbol{k}_g^{\sim *}(\omega)\, \boldsymbol{S}g^{\sim}(\omega)\, \boldsymbol{k}_g^{\sim T}(\omega) \tag{4.25}$$

Based on the equation (3.4), the autocorrelation functions corresponding to each of the subspaces S_x, S_y, S_θ and S_v will be computed according to the equation

$$b\,[w_a;\, t_n] = = \int_{-\infty}^{\infty} exp(\mathrm{i}\omega_m t_n)\, s\,[w_a;\, \omega_m]\, \mathrm{d}\omega_m \tag{4.26}$$

5. OBJECTIVES, OR STRATEGIES, OF ENGINEERING ANALYSES

The objective of this section is represented by a brief discussion on the goals of engineering analyses. This discussion relies on the developments of previous sections. It is intended to provide a connection between those developments and the possible alternative philosophies adopted by analysts of structural safety. In order to cover the possible alternative orientations, it is assumed at this place that the representation of seismic action adopted in order to conduct engineering analyses is that of direct use of accelerograms, which was referred to in subsection 3.2 as R.Ac..

A different application is related to the development of the Annex C of the….

A starting question: why are engineering analyses conducted? Of course, there are several possible answers to this question, and this depends first of all on the professional education and philosophy of the replying person. A brief attempt of reply is made as follows: keeping in view the current state of the art of engineering concepts concerning structural safety and reliability, it is possible to identify four alternative basic categories of objectives, or strategies, of increasing sophistication, of engineering calculations (Sandi, 2006). They are, respectively:

S.1: random computational experiment;

S.2: examination of sensitivity of output with respect to the variation of some (global, or macroscopic) input parameters;

S.3: analysis of seismic vulnerability;

S.4: full analysis of risk of exceedance of various limit states (in dependence of the duration of exposure, or of service).

The strategy S.1 may be witnessed indeed in most cases of engineering practice. As an example, analysts take often as input accelerograms some natural accelerograms recorded during some major, relevant, earthquakes (obtained at sites that are believed to be <but in reality may be more or less> relevant for a site dealt with), having more or less different spectral characteristics, durations etc.. The outcome of computations may put to evidence a sequence of development of plastic zones, some maximum displacements or drifts etc., some weaknesses of structures may be put to evidence etc..

The strategy S.2 may be used in order to reveal the sensitivity of structural response to some possible variation of (macroscopic) input parameters like amplitude, dominant frequencies, duration etc., and may provide in this way more relevant / complete information to design engineers.

The strategy S.3 comes up with a qualitative change and with increased requirements for the specification of seismic input. It should require generation of a set of sample accelerograms corresponding to a definite stochastic ground motion model and performing of analyses in the spirit of a Monte-Carlo approach (of course, the ground notion model, or alternative models, adopted will be believed to be relevant for a site dealt with). The computations should be performed repeatedly, for:

- the different sample accelerograms corresponding to a definite (macroscopic) calibration of the stochastic model of ground motion, times

- the different calibrations of macroscopic parameters characterizing the amplitude of motion, its spectral content and its duration, in a way to cover the domain of the space of macroscopic characteristics for which the occurrence of corresponding ground motions appears to be possible.

Finally, statistical analysis of results is to be carried out, in order to determine the distributions of the values of some relevant performance parameters or of damage grades, conditional upon the values of macroscopic parameters referred to. In case one counts the number of repeated runs required by such an approach, it is obvious that there is a huge increase of necessary runs as compared with the previous two strategies.

The strategy S.4 relies, as a basic step, on strategy S.3, which is to be followed primarily. On the other hand, it is necessary to characterize local hazard by means of appropriate characteristics of density of recurrence frequency in the space of macroscopic parameters (in case a Poissonian hazard model is accepted), or by means of other recurrence characteristics in case other, non-Poissonian, recurrence models are accepted (see, in this view, the developments of Section 2). Appropriate convolutions between hazard and vulnerability characteristics are to be finally performed, in order to reach desired, suitable, risk estimates. This is an extremely demanding approach and one can imagine its feasibility just in the frame of dedicated research projects.

A look at the codes in force reveals that they are still far from an approach in such terms. Would a code accept in the close future this categorization? Assuming the categorization adopted to be acceptable, new questions arise: how to decide about the use of one, or another, of the strategies referred to? It is to be recognized that passing from one of the strategies enumerated to a next one requires a considerable (if bearable) increase of skills required from the analyst, as well as of the volume of calculations, which means, of course, corresponding increase of time required for the analyses and of course, of costs. Suggestion: to accept nevertheless a categorization of this kind and to briefly describe the strategies, drawing the attention of code users to the technical advantages provided by the use of more sophisticated strategies.

6. FINAL CONSIDERATIONS AND COMMENTS

A few final comments on the topics presented above are related mainly to the relationship between the attempt of a consistent control of safety and risk on one hand and the constraints of feasibility for current practice, which lead to the adoption of pragmatic approaches relying on tradition and experience (by the way, it happens not very seldom that the belief in “experience” is denied by “surprises” that should have been expected if one kept in mind the consequences of a more consistent view on risk estimates).

A short look at the probabilistic analysis of structural safety and risk, as presented in Section 2, in a frame made as simple as possible, was used as a start point for subsequent developments. Care was given to emphasizing the simplifying assumptions which make possible such an approach. Relaxing the obstacles put to evidence for this approach would mean, at the same time, a possibility to make analyses of increasing relevance for the reality, but also an increase of sophistication of the analyses that can easily become unbearable from a pragmatic view point.

Some basic formal aspects of the representation and specification of seismic action and of seismic hazard were discussed in Section 3. They refer to the characterization of the expected earthquake ground motion during one event and, also, to the characterization of the expected sequence of seismic events. Again, care was given to emphasizing the simplifying assumptions accepted by the approaches that are usual in practice. The methodological implications of attempts at making these representations more relevant were dealt with too. Implementing the *n*D characterization of hazard is not an easy task, first of all due to the limits to basic information currently available. Perhaps a first step in this view would be a 2D approach, to account of the likelyhood of correlation between motion amplitude and dominant oscillation period.

After some references to the specific framework and to the simplifying assumptions were accepted, a stochastic model of the motion of ground, dealt with as a continuum, was presented. The model proposed provides the basic information for the explicit consideration of the simultaneous (non-synchronous) input along the various degrees of freedom of the ground – structure interface. This makes it possible to generate simultaneous (stationary) sample time histories of seismic action. For explicit consideration of the non-stationarity of motion, the canonic expansion (3.1) should be used. Of course, this model, based on postulation of some parameters, requires extensive additional studies and the main source of information of appropriate kind should be looked for in instrumental data provided by dense arrays.

After raising the question of the goal of engineering analyses, a short look to the alternative possible objectives of such analyses, as well as to the strategies and methodological implications of the choice between them was presented in Section 5. The acceptability for codes for practice of a more consistent view on safety control was discussed. It would be advisable to insert in codes at least some qualitative comments in this sense.

The topics tackled in the frame of the paper are of undeniable relevance and importance, from a theoretical point of view, for the improvement of the consistency of analysis of structural safety. It may be critically remarked that too little care was given to the feasibility of practical analyses according to the requirements formulated. The problem of the acceptability of such approaches for the codes for practice exists of course, but gradual steps for improving the consistency of codes should be undertaken nevertheless.

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