TRANSIENT HEAT TRANSFER AT THE BUILDING-GROUND BOUNDARY

Dan CONSTANTINESCU[*]

ABSTRACT

The heat flow-rate transferred through the ground represents an important rate of the heat flow-rate dissipated to the outside by the buildings, mainly by the low ones. The buildings architectural solutions associated to the structural solutions, meant to carry out buildings with a low energy consumption used by the thermal utilities (heating / cooling) emphasize the impact of the building – ground boundary on the building thermal response. This article presents a hybrid calculation model of the ground thermal response to the indoor and outdoor random thermal loads, based on the combining the model specific to the steady-state solution by the conforming transformations method used in the case of the previously mentioned boundary and the Unitary Thermal Response (RTU) method specific to the impulse thermokinetics used in the case of virtual similar environments operating as a model, which substitutes the real environment (ground).

The article also presents the results of the experiments performed on the CE INCERC Bucharest experimental building in the cold seasons 2003 and 2004. The deviations between the heat flow-rate experimental values and those theoretically determined by the INVAR software, ranging between 2.48 % and 5.45 % certify the proposed calculation model. The practical use of the calculation model is meant for the boundaries between the building and ground in several variants; it is included as a calculation method in the Romanian EPB (building energy performance) calculation method, Mc 001 / 1-2006.

Key-words: conform transformation, virtual outdoor temperature, similitude criteria, hybrid mathematical model, boundary conditions, homogeneous and non-homogeneous environments

REZUMAT

Fluxul termic transferat prin sol reprezintă o cotă importantă din fluxul termic disipat către exterior la nivelul construcțiilor, în special a celor cu nivel redus de înălțime. Rezolvările de arhitectură asociate cu rezolvările structurale ale clădirilor, care vizează realizarea unor clădiri cu consum redus de energie, aferent utilităților termice (încălzire / răcire), aduc în prim plan impactul frontierei clădire-sol asupra bilanțului termic al clădirii. Lucrarea de față prezintă un model de clacul hibrid al răspunsului termic al solului la solicitările termice aleatoare interioare şi exterioare, bazat pe combinația modelului propriu rezolvării în regim stationar prin metoda transformărilor conforme aplicată frontierei susmenționată, şi metoda Răspunsului Termic Unitar (RTU), proprie termocineticii impulsionale aplicată mediilor similare virtuale cu funcție de model, care substituie mediu real (solul).

Se prezintă rezultatele unor experimentări realizate pe suportul clădirii experimentale CE INCERC Bucureşti în sezoanele reci 2003 şi 2004. Abaterile între valorile experimentale de flux termic şi cele determinate teoretic prin utlizarea programului de calcul INVAR, cuprinse între 2,48 % şi 5,45 %, certifică modelul de calcul propus. Aplicarea practică a modelului de calcul se adresează frontierelor dintre clădire şi sol în diferite variante de realizare şi este inclusî ca metodă de calcul în metodologia autohtonă de calcul al PEC, Mc 001/ 1 – 2006.

Cuvinte cheie: transformare conformă, temperatură exterioară virtuală, criterii de similitudine, model matematic hibrid, condiții la limită, medii omogene şi neomogene

[*)] INCD URBAN-INCERC, INCERC Bucharest Branch, Romania; e-mail: dan.constantinescu@incerc2004.ro

1. GENERAL PRESENTATION

The heat flow-rate transferred through ground represents an important rate of the heat flow-rate dissipated to the outside at the level of buildings, mainly of those that are not too high. The literature in the field estimates that for such a building, properly insulated in the upper part (namely over the ground level) and which has components in contact with the ground which are not insulated, the heat flow transferred through the ground is 60 % of the whole flow yielded to the environments.

Most of the calculation methods deal with the flow through the ground in an improper manner, using the steady-state or quasi-steady-state heat transfer conditions. The heat transfer steady-state conditions might be considered only if the ground layer thermal capacity is very low and the temperatures of the dwelled and outdoor environments are constant for long periods of time (which is not the case). The indoor temperature varies in time in the case of dwelled spaces equipped or not with continuously operating air conditioning systems. The outdoor temperature has both daily and seasonal variations; moreover, the solar radiation influence must be added here. Therefore, this is another parameter which does not meet the conditions of steady-state heat transfer through the ground. The calculation methods used in Romania do not take into consideration this temperature variation of the environments, between which the heat flow-rate occurs, or the phase lag or the damping of the thermal waves generated by the ground thermal capacity. The proper use of certain transient heat transfer models leads to technical solutions of building components thermal configuration as well as to values of the occupied spaces heat demand which vary in time; both have consequences in the economic sector as well as in the operation of buildings and their corresponding systems.

This article presents a hybrid method of transient heat transfer through ground, based on the analysis of the heat transfer and of the spectrum of isotherms specific to the transient conditions [1, 2] associated to the virtual outdoor temperature of the adjoining natural environment, which varies both in time and according to the heat flow-rate lines length. The calculation model of the thermal flows through ground uses the geometry of the flow lines specific to the steady-state conditions and is supported by the transient heat transfer through flow channels. Actually the heat flow-rate lines are assimilated to tubular lines of flow, generating flow channels which are infinitely thin and with a length equal to the flow line, which connects the point on the room floor to the outdoor environment. The material structure of the flow channel contains all the material structures included in the floor and ground structures. The flow channels are displayed on the entire area adjoining the ground on the outside of the building component. Such adjoining flow channels, each infinitely thin, are characterized only by heat transfer along them as, consequent to the reduced temperature difference, the radial heat transfer is considered negligible. A simplified model results therefore, characterized by one-dimensional heat transfer along the flow channels; the adjoining areas of two successive flow channels are adiabatic areas.

In order to use the Unitary Thermal Response Method (RTU), the criteria of similitude between the real process and the model are used. The transient heat transfer method was validated by comparing the results provided by this calculation model to those provided by the programme of experiments performed on an INCERC Bucharest testing building; a number of measurements were performed here in the 2003-2004 winter, focused on the heat flow-rate dissipated on the floor. These measurements were performed in two rooms, one West directed and the other North-West directed (the directions represent marks of the main facades).

The comparison of the measured and calculated values proved errors ranging between 0.6 % and 6.23 %. The use of a method described in the Romanian literature in the field area, in the same case, generated values more than twice higher for the thermal flow, therefore a very important deviation compared to reality (100 %). This value can be explained by the use of the steady-state heat transfer, which correlates the indoor and the outdoor temperatures, a hypothesis which is entirely contradicted by the analysis of the specific heat flow-rate measured values.

The practical importance of the new calculation method is the possibility of optimizing the costs of the protection of the floors placed on ground by accurately knowing the variation of the values specific to the heat flow dissipated through ground as well as a new approach of the works of upgrading existing buildings. The thermal system dimensioning and the heat consumption are equally influenced by the accurate quantification of the heat flow dissipated through ground.

The calculation method is based on a common simplification of all the calculation models used in assessing the Buildings Energy Performance (EPB), namely one-phase heat transfer, but expressly considering the thermophysical parameters specific to humid ground, dry or frozen according to the ground water content and to the thermal conditions specific to the heat flow-rate lines virtual paths. In the case of processes characterized by phase change (e.g. ground frost / thaw), the model takes into account the thermophysical properties of ground following the previously mentioned processes (equivalent thermal conductivity consequent to the use of Stefan equation).

2. IMPULSIONAL THERMOKINETIC ELEMENTS – THE *UNITARY THERMAL RESPONSE* METHOD (RTU)

The mathematical modeling observing the impulse thermokinetics is used in order to solve the problems involving variable parameters and random climatic loads. The thermal response based on steady-state heat transfer in 2D space, one of the hypotheses extensively used in modeling the heat transfer through [1, 2] and which benefits from the conform depiction method, leads to the determination of the heat-flow rate curves in the form of ellipse segments perpendicular on the isothermal curves. The heat flow-rate "lines" may be assimilated to a very good approximation of circle arcs, which offer the possibility of providing a number of practical solutions used in the calculation methods specific to the building-ground boundary heat transfer. The hypothesis of steady-state heat transfer through ground, impossible to argument phenomenologically, has the advantage to facilitate algebraic solutions easily usable and highly versatile from the operational point of view, which is a considerable advantage in engineering calculations.

But the lack of phenomenological conformity abolishes the qualities to be considered in using the solutions specific to the heat transfer steady-state conditions. A compromising solution between the previously described simplification, specific to the steady-state solution and the solution specific to the real conditions, variable in terms of heat transfer involves the use of the virtual outdoor temperature specific to the outdoor environment instead of the outdoor environment temperature.

This temperature is a function variable in terms of space which is represented by the variable lengths of the heat flow-rate lines and in terms of time, a consequence of the transfer function specific to the ground massive defined between the inside thermodynamic outline (adjoining the building) and the outside thermodynamic outline (adjoining the natural or built outdoor environment). The theory of the virtual outdoor temperature defined as a space and time function, $t_{ev}(s,\tau)$ [3] proves that it is allowed to use the mathematical formalism specific to the steady-state heat transfer by conduction in transient heat transfer problems. Based on solution $q_i(\tau)$ – heat flow density value at level $x = 0$ – a very important thermodynamic parameter may be defined, which will be further used in this article, namely the *virtual outdoor temperature*.

Temperature $t_{ev}(\tau)$ represents *a virtual temperature of the outdoor environment in terms of which, at any moment, the heat flow-rate dissipated to / coming from the outdoor environment may be determined by the mathematical formalism specific to the steady-state heat transfer*.

The main characteristic of the virtual outdoor temperature is *its invariance in terms of the variation of the indoor temperature*, t_i. According to the *impulse thermokinetics* method, the solution of the conduction equation is based on the *thermal response* in the form of thermal flows and field of temperatures in flat and isotropic structures, at impulsive outdoor loads. An impulsive load is a sudden modification of a climatic parameter, which is also accompanied by an energy impulse applied on the flat component under analysis. The heat parabolic equation in one-phase and isotropic environments, if indoor heat sources are lacking is a *linear* equation with partial derivatives. The thermal response has the form of a variable function in terms of time as the result of the linear system loading by an impulse-type excitation. The outlet function $y\,(\tau)$ is the consequence of using the inlet function $z\,(\tau)$; the link between the two functions is performed by means of the *Unitary Thermal Response (RTU)* function [3]:

$$y\,(t) = z\,(t) * r\,(t) \tag{6}$$

where (*) represents the convolution product of functions $z\,(t)$ and $r\,(t)$.

Function $r\,(t)$ represents the *Unitary Thermal Response (RTU)* as it is an outlet function of the linear system characterized by the $\psi(\tau)$ transfer function under Dirac impulsive load.

The thermal response is represented by the value of the specific heat flow-rate at levels $x = 0$ and $x = \Delta\,(r_1\,(\tau))$, namely $r_2\,(\tau)$.

$$r_1(\tau) \equiv X\,(\tau) \tag{7}$$

$$r_2(\tau) \equiv Y\,(\tau) \tag{8}$$

Therefore the final solutions of the heat conduction problem with Dirichlet boundary conditions are written as follows:

$$\begin{pmatrix} \{q_i(\tau)\} \\ \{q_e(\tau)\} \end{pmatrix} = \begin{pmatrix} \{X\,(\tau)\}^T & \{Y\,(\tau)\}^T \\ \{Y\,(\tau)\}^T & \{X\,(\tau)\}^T \end{pmatrix} \cdot \begin{pmatrix} \{\varphi_1(\tau)\} \\ \{\varphi_2(\tau)\} \end{pmatrix} \tag{11}$$

where: $\{\ \}^T$ – line vector; $\{\ \}$ – column vector

The result is that the solution of any conduction heat transfer problem is reduced to knowing the value of the line matrix elements $\{X\,(\tau)\}^T$ and $\{Y\,(\tau)\}^T$, these values representing exactly the *Unitary Thermal Response*.

In the case of a multi-layer flat structure, the problem is completed by the IV-th rank boundary conditions at the contact between the material structures characterized by a Δ_j thickness, λ_j thermal conductivities, ρ_j densities and c_j mass specific heat as well as by the III-rd rank boundary conditions at the contact with the adjoining outdoor environments. It results that for a flat environment formed of n material structures, two equations will be written, representing the IV-th rank boundary conditions. A system of n equations with n unknown values is generated; the latter items represent the temperatures on the surfaces bordering the material layers. According to the III-rd rank boundary conditions, linear or not, a linear or non-linear system of algebraic equations results, which is solved by specific methods. The thermal balance of a multi-layer structure with n material layers is written in the form of the following equations:

$$\begin{cases} \alpha_i[t_i(\tau) - t_{Pi}(\tau)] - [\sum\limits_{k=0}^{m} X_1(\tau - k\Delta\tau)\cdot t_{Pi}(k\Delta\tau) + \\ + \sum\limits_{k=0}^{m} Y_1(\tau - k\Delta\tau)\cdot \vartheta_1(k\Delta\tau)] = 0 \\ \sum\limits_{k=0}^{m} X_1(\tau - k\Delta\tau)\cdot \vartheta_1(k\Delta\tau) + \sum\limits_{k=0}^{m} Y_1(\tau - k\Delta\tau)\cdot t_{Pi}(k\Delta\tau) = \\ = \sum\limits_{k=0}^{m} X_2(\tau - k\Delta\tau)\cdot \vartheta_1(k\Delta\tau) + \sum\limits_{k=0}^{m} Y_2(\tau - k\Delta\tau)\cdot \vartheta_2(k\Delta\tau) \\ \sum\limits_{k=0}^{m} X_2(\tau - k\Delta\tau)\cdot \vartheta_2(k\Delta\tau) + \sum\limits_{k=0}^{m} Y_2(\tau - k\Delta\tau)\cdot \vartheta_1(k\Delta\tau) = \\ = \sum\limits_{k=0}^{m} X_3(\tau - k\Delta\tau)\cdot \vartheta_2(k\Delta\tau) + \sum\limits_{k=0}^{m} Y_3(\tau - k\Delta\tau)\cdot \vartheta_3(k\Delta\tau) \\ \ldots\ldots. \\ \sum\limits_{k=0}^{m} X_n(\tau - k\Delta\tau)\cdot t_{Pe}(k\Delta\tau) + \sum\limits_{k=0}^{m} Y_n(\tau - k\Delta\tau)\cdot \vartheta_{n-1}(k\Delta\tau) + \\ + \alpha_e\cdot[t_{Pe}(\tau) - t_E(\tau)] = 0 \end{cases} \tag{12}$$

the unknown values of which are $t_{Pi}\,(\tau)$, $\vartheta_1(\tau)$, $\vartheta_2(\tau)$, ..., $\vartheta_{n-1}(\tau)$, $t_{Pe}(\tau)$.

If the building materials thermal conductivity varies according to the temperature, any structure may be divided into an infinite number of layers. The solution is obtained based on the method that has been presented, by iterative calculation; its indicator is the convergence of the temperature values in each layer, in terms of the law on the variation of the thermal conductivity according to the temperature.

A useful characteristic of RTU is provided by the following relations:

$$\begin{cases} \sum\limits_{k=0}^{m} X\,(\tau - k\Delta\tau) + \sum\limits_{k=0}^{m} Y\,(\tau - k\Delta\tau) = 0 \\ \left|\sum\limits_{k=0}^{m} X\,(\tau - k\Delta\tau)\right| = \left|\sum\limits_{k=0}^{m} Y\,(\tau - k\Delta\tau)\right| = \dfrac{\lambda}{\Delta} \\ \sum\limits_{k=0}^{m} X\,(\tau - k\Delta\tau) > 0 \end{cases} \tag{13}$$

Based on the Unitary Thermal Response (RTU) theory, the INVAR software was conceived [4], aimed to determine the heat / cold demand of dwelled / occupied spaces as well as the variation of the significant indoor temperatures in the dwelled / occupied spaces both in the cold and in the hot season.

Figures 1…3 present graphically the unitary thermal responses $X(\tau)$ and $Y(\tau)$ of a 0.25 m thick reinforced concrete plate, a material frequently included in the structure of opaque closing components. In order to validate the calculation method, the results obtained by the RTU method (analytical) were compared to those provided by the numerical software by the finite element method – ANSYS, a numerical analysis software based on the finite element method, considered a reference software in the field of mechanical and thermal simulations in dynamic or steady-state conditions.

Figure 2 presents the temperature variation on the inside and outside surfaces of the plate. A very good correlation is noticed between temperatures $t_{pi}(\tau)$ and $t_{pe}(\tau)$ provided by ANSYS and the temperatures determined analytically (convolution).

Verification:

$$\sum_{i=0}^{n} X = -\sum_{i=0}^{n} Y = \frac{\lambda}{\delta}.$$

δ/λ numerical	δ/λ real (analytical)	Error [%]
0.14367952	0.143678161	0.00094578

Analytical: Unitary heat flow-rate integrated for 24 h stabilized conditions [kWh / m²]: 1.28411

Numerical: Unitary heat flow-rate integrated for 24 h stabilized conditions [kWh / m²]: 1.28912

Deviation: [%]: 0.39000

The diagram in fig. 3 presents the variation of the outdoor temperature and of the virtual outdoor temperature during 24 h of thermal loading. The virtual outdoor temperature was determined based on the variation of the density of the heat flow-rate dissipated at level $x = 0$, both by the convolution analytical method and by the two software: INVAR and ANSYS. The results prove to be properly close, which is a mutual validation of the analytical and numerical calculation methods.

Moreover, the variation in time of the virtual outdoor temperature clearly emphasizes the damping and phase lagging properties of the thermal wave of the analyzed structure (in this case the 25 cm thick reinforced concrete plate). It is noticed that, unlike the results provided by the analysis of the

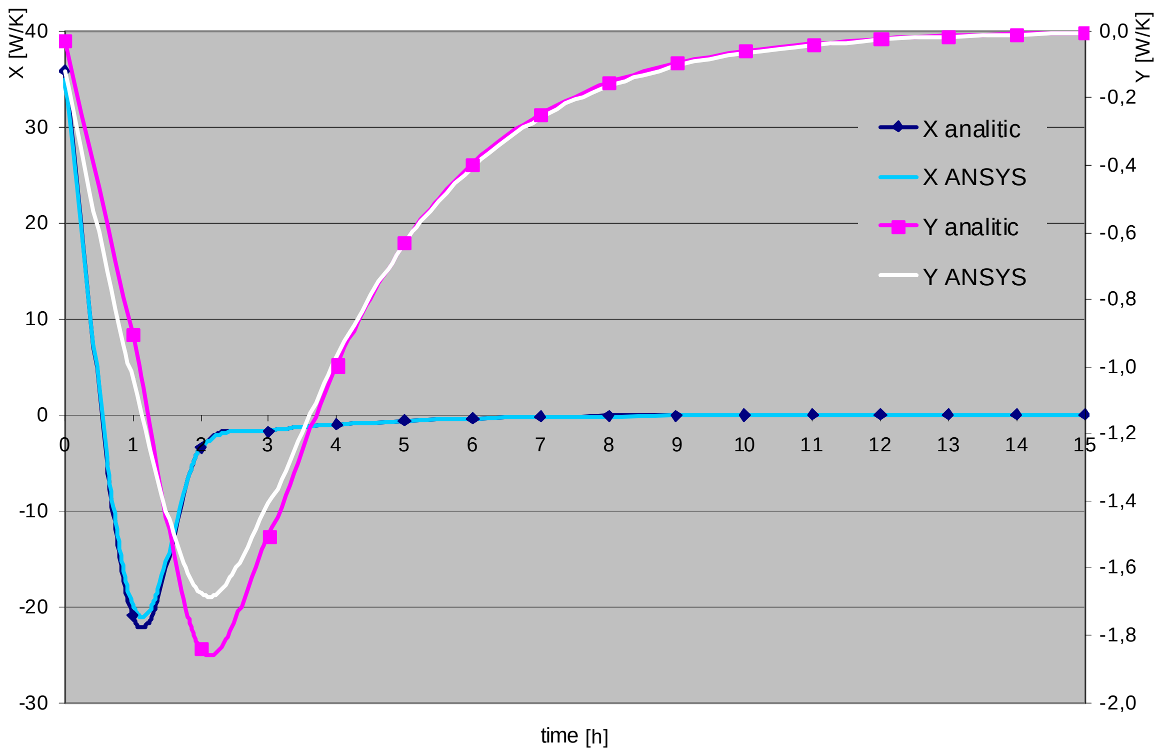

Fig. 1. Unitary Thermal Response of structure 1

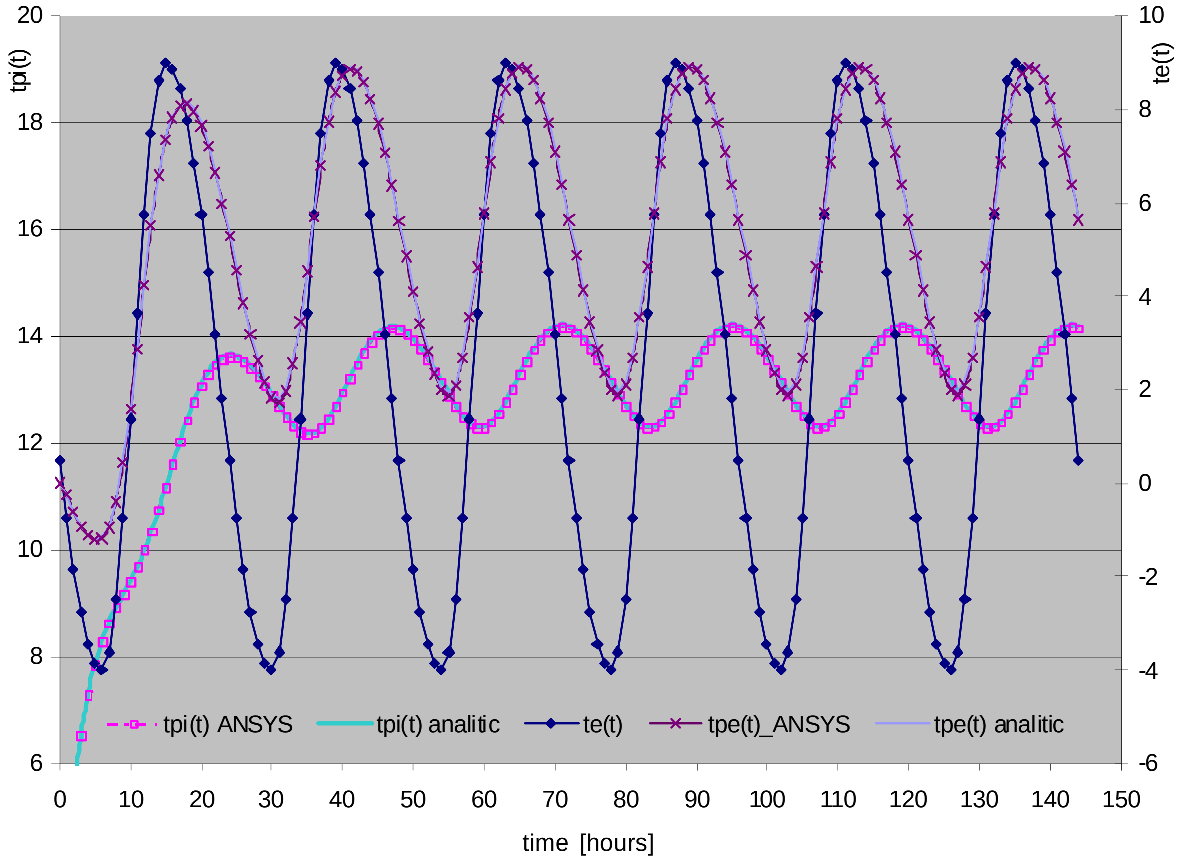

Fig. 2. Variation of the temperature of the flat plate surface (t_{int} = 20°C)

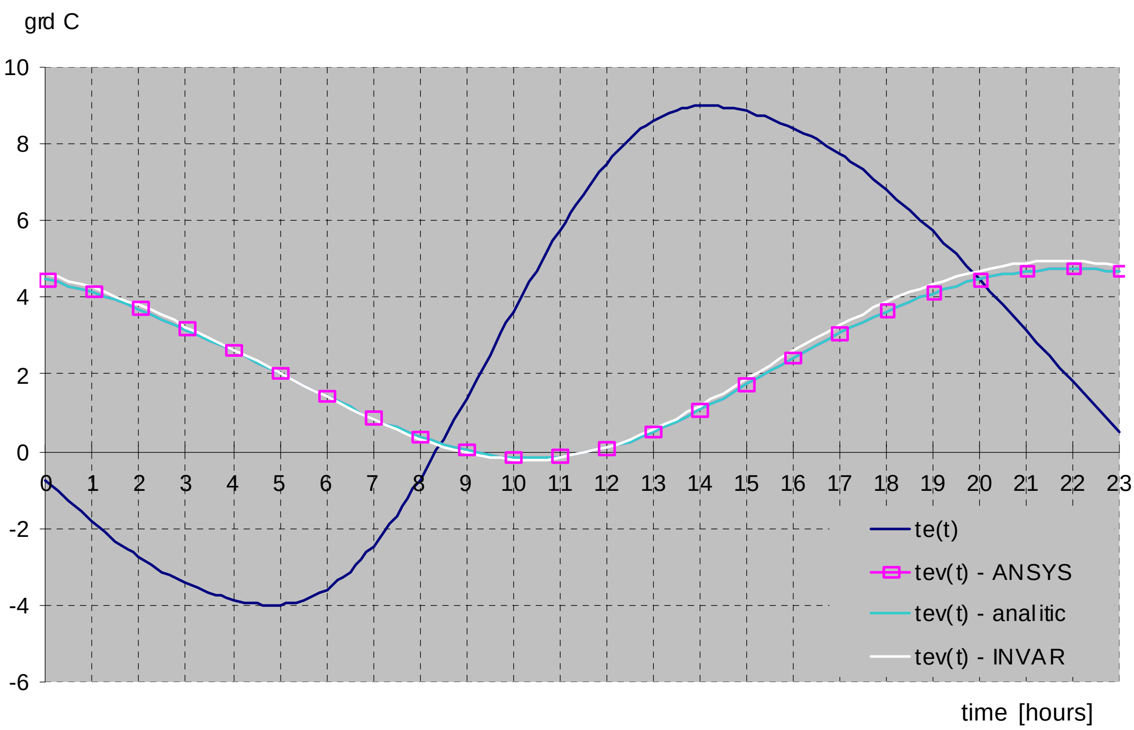

Fig. 3. Virtual outdoor temperatures variation in terms of outdoor temperature

harmonic functions as excitation and response functions, the RTU based analysis emphasizes different phase lags for the temperatures minimum values (5.5 h) and for the temperatures maximum values (7.5 h).

According to the relations specific to the heat transfer steady-state conditions, if the indoor temperature variation in time, $t_i\,(\tau)$ is known as well as the virtual outdoor temperature variation, $t_{ev}\,(\tau)$ and the building component thermal resistance, the heat flow-rate of the area adjoining the indoor environment is determined by the following relation:

$$Q_i(\tau) = \frac{S}{R_1} \cdot [t_i(\tau) - t_{ev}(\tau)] \tag{14}$$

where $t_{ev}\,(\tau)$ is determined based on value $q_i\,(\tau)$, using the following relation:

$$t_{ev}(\tau) = t_{i0} - R \cdot q_i(\tau) \tag{15}$$

where $q_i\,(\tau)$ is determined by relations (12) according to the temperature values resulted by solving the above mentioned system.

3. SIMILITUDE CRITERIA

The use of the INVAR software in the case of massive multi-layer structures implies significant calculation time. The working time is reduced, without altering the result accuracy, by using virtual calculation structures of the "model" type, the thermophysical characteristics of which are determined by using the similitude criteria.

The following equation is considered representative for the model (m) and for reality (r):

$$\frac{\partial \vartheta_m}{\partial \tau_m} = a_m \cdot \frac{\partial^2 \vartheta_m}{\partial x_m^2} \tag{16}$$

and:

$$\frac{\partial \vartheta_r}{\partial \tau_r} = a_r \cdot \frac{\partial^2 \vartheta_r}{\partial x_r^2} \tag{17}$$

Any of the equations (16) and (17) is integrated in a field $x \in [0, \Delta]$, defined by the flat plate thickness. The dimensionless thickness of the plate is defined:

$$\tilde{x} = \frac{x}{\Delta} \tag{18}$$

Consequently, the equations become:

$$\frac{\partial \vartheta_m}{\partial \tau_m} = \frac{a_m}{\Delta_m^2} \cdot \frac{\partial^2 \vartheta}{\partial \tilde{x}^2} \tag{19}$$

$$\frac{\partial \vartheta_r}{\partial \tau_r} = \frac{a_r}{\Delta_r^2} \cdot \frac{\partial^2 \vartheta_r}{\partial \tilde{x}^2} \tag{20}$$

The fundamental condition of using equation (19) instead of equation (20) is that the temperature should result in the same form $\vartheta\,(\tilde{x}, \tau)$ and with the same values both in the model case and in the real process case. The use of modeling by similitude is useful mainly in two cases, as follows:

- in the case of numerical modeling of the building components with a very high thermal capacity (ground), when the solution based on the real structure requires a large quantity of memory;
- in the case of preparing "in vitro" experiments.

Each of the equations (19) and (20) are written in the following form:

$$\frac{\partial \vartheta_m}{\partial Fo_m} = \frac{\partial^2 \vartheta_m}{\partial \tilde{x}^2} \tag{21}$$

$$\frac{\partial \vartheta_r}{\partial Fo_r} = \frac{\partial^2 \vartheta_r}{\partial \tilde{x}^2} \tag{22}$$

Taking into account the remark on the temperature field solution, it results that the following condition is imposed:

$$\mathrm{Fo}_m = \mathrm{Fo}_r \tag{23}$$

where Fo is Fourier dimensionless number defined according to time, material thermal diffusivity flat plate thickness.

The solution $\vartheta\,(x, \tau)$ is obtained by attaching the boundary conditions to the heat parabolic differential equation. The case of a flat plate and of a multi-layer flat structure are further considered.

a. In the case of a flat plate, the III-rd rank boundary conditions, which connect the plate adjoining environments and the plate lead to two conditions, each of them specific to the adjoining environments contact boundary:

$$\mathrm{Bi}_m \big|_{\tilde{x}=0} = \mathrm{Bi}_r \big|_{\tilde{x}=0} \tag{28}$$

$$\mathrm{Bi}_m \big|_{\tilde{x}=1} = \mathrm{Bi}_r \big|_{\tilde{x}=1} \tag{29}$$

Equation (23) becomes:

$$\frac{\lambda_r \tau_r}{(\rho c)_r \cdot \Delta_r^2} = \frac{\lambda_m \tau_m}{(\rho c)_m \cdot \Delta_m^2} \tag{30}$$

If a R_τ value is imposed to the ratio:

$$R_\tau = \frac{\tau_r}{\tau_m} >> 1 \tag{31}$$

the time used in the model is "compressed" compared to reality. Equation (30) becomes:

$$\left(\frac{\Delta_m}{\Delta_r}\right)^2 \cdot \frac{(\rho c)_m}{(\rho c)_r} \cdot \frac{\lambda_m}{\lambda_r} \cdot R_\tau = 1 \tag{32}$$

Equations (28) and (29) consequently become:

$$\frac{\alpha_{im}\Delta_m}{\lambda_m} = \frac{\alpha_{ir}\Delta_r}{\lambda_r}$$

$$\frac{\alpha_{em}\Delta_m}{\lambda_m} = \frac{\alpha_{er}\Delta_r}{\lambda_r}$$

Conditioned by $\alpha_{im} = \alpha_{ir}$, $\alpha_{em} = \alpha_{er,}$ the result will be:

$$\frac{\Delta_m}{\Delta_r} \cdot \frac{\lambda_r}{\lambda_m} = 1 \tag{33}$$

By introducing equation (31) in (33), the value of the thermal conductivity of the model material is obtained:

$$\lambda_m = \lambda_r \cdot \frac{(\rho c)_r}{(\rho c)_m} \cdot R_\tau^{-1} \tag{34}$$

The second important unknown value is obtained as well:

$$\Delta_m = \Delta_r \cdot \frac{(\rho c)_r}{(\rho c)_m} \cdot R_\tau^{-1} \tag{35}$$

Based on values λ_m and Δ_m, the model similar to reality in terms of phenomenology can be built.

b. The second case involves n plates in thermal contact; the whole structure adjoins the environments at temperatures α_i and α_e. Taking into account the elements above, the following equations are obtained:

$$\begin{cases} \left[\left(\dfrac{\Delta_m}{\Delta_r}\right)^2 \cdot \dfrac{(\rho c)_m}{(\rho c)_r} \cdot \dfrac{\lambda_m}{\lambda_r}\right]_{(1)} \cdot R_\tau^{-1} = 1 \\ \dots \\ \left[\left(\dfrac{\Delta_m}{\Delta_r}\right)^2 \cdot \dfrac{(\rho c)_m}{(\rho c)_r} \cdot \dfrac{\lambda_m}{\lambda_r}\right]_{(n)} \cdot R_\tau^{-1} = 1 \end{cases} \tag{36}$$

to which the III-rd rank boundary conditions (33) are attached:

$$\begin{cases} \left[\left(\dfrac{\Delta_m}{\Delta_r}\right) \cdot \dfrac{\lambda_r}{\lambda_m}\right]_{(1)} = 1 \\ \dots \\ \left[\left(\dfrac{\Delta_m}{\Delta_r}\right) \cdot \dfrac{\lambda_r}{\lambda_m}\right]_{(n)} = 1 \end{cases} \tag{37}$$

The IV-th rank boundary condition, specific to thermal contact problems, is imposed for the succession of layers (1), ..., (n), in the following form:

$$\left[\left(\frac{\Delta_m}{\Delta_r}\right) \cdot \frac{\lambda_r}{\lambda_m}\right]_{(1)} = \left[\left(\frac{\Delta_m}{\Delta_r}\right) \cdot \frac{\lambda_r}{\lambda_m}\right]_{(2)} = $$

$$= \dots = \left[\left(\frac{\Delta_m}{\Delta_r}\right) \cdot \frac{\lambda_r}{\lambda_m}\right]_{(n)} \tag{38}$$

By associating system (37) to system (38), a number of $2n$ equations with $2n$ unknown values results, namely $\Delta_{m\,(1)}$, ..., $\Delta_{m\,(n)}$ and $\lambda_{m\,(1)}$, ..., $\lambda_{m\,(n)}$. The solutions resulted are similar to solutions (34) and (35):

$$\lambda_{m(j)} = \lambda_{r(j)} \cdot \left[\frac{(\rho c)_r}{(\rho c)_m}\right]_{(j)} \cdot R_\tau^{-1} \tag{39}$$

$$\Delta_{m(j)} = \Delta_{r(j)} \cdot \left[\frac{(\rho c)_r}{(\rho c)_m}\right]_{(j)} \cdot R_\tau^{-1} \tag{40}$$

where $j \in [1, n]$.

It is noticed that the model dimensioning depends on the ρc ratios *the values of which may be arbitrary*.

Two numerical examples are further presented, one represented by an extremely massive structure of 2 m thick stone and the other a massive tri-layer structure made, from the outside to the inside, of 1 m thick brick, 0.01 m thick thermal insulation (expanded polystyrene) and 0.20 m thick reinforced concrete.

Value $R_{\tau(1)}$ = 360 is adopted for the first case, which means that a time-lag of 1 h in the model is equivalent to 360 h in reality (15 days). Therefore a simulation of 24 h on the model is equivalent to the thermal response for a period of 12 months (1 year).

In the first case, the real characteristics are:

Δ_r = 2.00 m

λ_r = 2.55 W / mK

ρ_r = 2,420 kg / m^3

c_r = 920 J / kgK

The characteristics of the model become:

$$\Delta_m = 0{,}002778 \cdot \frac{(\rho c)_r}{(\rho c)_m} \cdot \Delta_r$$

$$\lambda_m = 0{,}002778 \cdot \frac{(\rho c)_r}{(\rho c)_m} \cdot \lambda_r$$

If the arbitrary values are chosen, ρ_m = = 70 kg / m^3 and c_m = 1,460 J / kgK, the result is:

Δ_m = 0.12 m

λ_m = 0.055 W / mK

(thermophysical characteristics of polyvinyl chloride foam).[1)]

The procedure is similar in the second case. The results are presented in Table 2, mentioning that $R^{-1}_{\tau(2)} = 0.125$.

The most useful applications are on buildings with extremely massive envelope (e.g. churches) and on heat transfer through ground.

4. RESULTS OBTAINED ON THE SUPPORT OF THE INCERC EXPERIMENTAL BUILDING

Within the experimental programme carried out in the INCERC Bucharest test building in the season 2003-2004, a number of measurements were performed on the heat flow-rate dissipated at the level of the floor. The measurements were performed in two rooms, one West-directed and the other North-West directed (the directions represent marks of the main facades). We mention that the inside of the test building was preserved in the cold season at a thoroughly controlled indoor temperature and was continuously monitored. In order to provide objective results, the perturbing influence of human activity was minimized.

4.1. Description of the measurement chain

The long-term usual measurements performed in the INCERC experimental building in the cold season 2003-2004 allowed us to obtain the following values:

- $Pe(\tau)$ – electric power at the energy source [W];
- $E(\tau)$ – electric power consumed between two successive readings at the level of the energy source [kWh];
- $G_S(\tau)$ – heat carrier volume flow-rate at the level of the heating energy source [m^3/h];
- $t_{ac1}(\tau)$ – indoor temperature of the living (dining) room air [°C];

Table 2.

Characteristics of the model and of the real structure

Layer	Δ_r	λ_r	ρ_r	c_r	Δ_m	λ_m	ρ_m	c_m
1	1.00	0.800	1,800	870	0.125	0.1000	1.800	870
2	0.10	0.045	20	1.460	0.007	0.0021	70	750
3	0.20	1.740	2,500	840	0.100	0.8700	700	750

1) If a physical model is performed, λ_m, ρ_m and c_m should characterize a material.

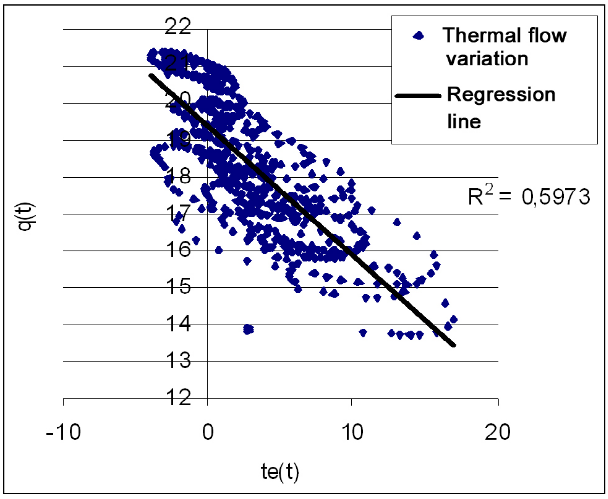

Fig. 7. Correlation dissipated heat flow-rate – outdoor temperature for the outline strip. Correlation heat flow-rate – t_e for the outline strip

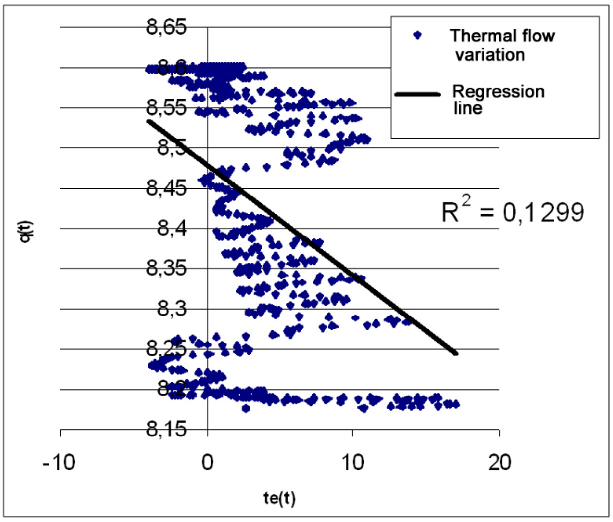

Fig. 8. Correlation dissipated heat flow-rate – outdoor temperature for the central zone of the flat plate on ground. Correlation heat flow-rate – te for the central zone

Table 3.

R^2 correlation rate specific to the linear regression heat flow-rate – outdoor temperature for 5 cm wide strips at different levels on the thermodynamic outline

Strip located between the levels as against the plate edge [cm]	R^2 correlation rate
0-5	0.9200
5-10	0.7616
10-15	0.6360
15-20	0.5269
20-25	0.4472
25-30	0.3800

$t_{ac2}(\tau)$ – indoor temperature of the NW bedroom air [°C];

$t_{ac3}(\tau)$ – indoor temperature of the NE bedroom air [°C];

$t_{ac4}(\tau)$ – indoor temperature of the kitchen air [°C];

$t_e(\tau)$ – outdoor air temperature [°C];

$I_G, I_d(\tau)$ – intensity of global, respectively diffusive solar radiation [W / m²];

$t_{g1}(\tau)$, $t_{g2}(\tau)$ – globe thermometers temperature [°C].

The equipment used for the measurements consists in the following items:

- *Aquametro* turbine flow-meter, PMG Dn 32, with RH impulse transmitter;
- TPM-79 electronic transducer for electric power measuring;
- *Sauter* EGT 420 indoor air temperature probes with PT 100 sensors, ± 0.12 % error – for measuring temperatures t_{ac1}, t_{ac2}, t_{ac3} and t_{ac4};
- *Sauter* EGT 300 outdoor air temperature probe with Ni 1000 sensor, ± 0.12 % error – for measuring temperature t_e;
- CMP 6 pyranometer – for measuring global solar radiation and the solar radiation diffuse component;
- DataTaker 50 automatic data recorder with a processor.

The heat flow-rate dissipated through the building components in contact with ground was measured with a calibrated flow-meter strip. The heat flow-rate value was experimentally determined in two spots distinctly located at the level of the outline strip of the investigated rooms. One measurement point is located in the central zone of the outline strip and the second in its corner. The two positions are significant for the heat transfer to the outside.

For the outline strip (fig. 7) the correlation is only qualitative, as the calculated value of the correlation coefficient (R^2 = 0.5973) indicates the lack of a possibility to express the heat flow-rate in terms of the outdoor temperature in the form of a linear function, specific to the heat transfer steady-state conditions. For the central zone (fig. 8), located between a 1 m distance from the thermodynamic outline and the symmetry axis, the correlation coefficient is $R^2 = 0.1299$. The variation of the linear regression correlation degree, R^2, according to the level of the strip (as against the thermodynamic outline edge) is synthetically presented in table 3.

These values prove that the heat flow-rate dissipated through the building components in contact with ground *cannot be accurately determined using the heat transfer steady-state conditions*.

5. A MATHEMATICAL MODEL OF HEAT TRANSFER THORUGH GROUND IN TRANSIENT CONDITIONS – APPROXIMATE METHOD

The mathematical model is a hybrid model combining the structure of the heat flow-rate lines specific to the steady-state heat transfer by conduction on one hand and the Unitary Thermal Response (RTU) method, specific to the transient heat transfer by conduction, on the other. The model takes into account the variation in time of the indoor temperature. Actually, in the cold season, the indoor temperature is preserved at a constant value while in the hot season it marks a variation characterized by a maximum in July-August. Therefore the mathematical model of heat transfer through ground to the outdoor environment and to the water bed in the groundwater or to the constant temperature layer will take into account the variation $t_i(\tau)$.

Both analysis methods, to the outdoor environment and to the constant temperature layer, will use the RTU method by using the criteria of similitude between the Real process and the Model (item 3).

5.1. Heat transfer to the outdoor environment

1. The flow channel length is determined by the following relation:

$$\delta_R = h + \beta \cdot r$$

where: β – angle on the arc of circle uniting the point on the floor and the outside plane [radians]; r – arc of circle radius [m]; h – distance from the floor to the Systematized Land Level, SLL[m].

2. Values $R_\tau > 1$ are chosen in order to use $\delta_M < \delta_R$ in the calculation.

5.2. Heat transfer to the waterbed in the groundwater

Two ground layers the thickness of which frequently exceeds the value of $\delta_R = 4$ m are considered. The time scale single value of $R_\tau = 360$ is adopted. The outdoor environment of the plate is, in fact, the dwelled space characterized by variable temperatures during one year (the hourly value in the model is t_i average for a period of 15 consecutive days). The solar radiation intensity is null and the virtual wind velocity is selected so as to provide coefficient α_i specific to the heat transfer on the floor surface, according to Jurges relation [18]:

$$\alpha_{iPd} = a + bw \qquad (41)$$

which provides:

$$w = \frac{\alpha_{iPd} - a}{b}\ \text{[m/s]} \qquad (42)$$

which is further introduced in the “outdoor climate” file of the INVAR soft.

For the constant temperature layer assimilated to the environment where the temperature $t_{i0} = t_{groundwater}$ is preserved, values $\alpha_{cv} = \alpha_r > 200\ \text{W/m}^2\text{K}$ are chosen which actually generate a Dirichlet type condition (I-st rank) at the level of the constant temperature layer.

Following the running of the INVAR soft, values $q_{ext}(\tau)$ and $t_{Pd}(\tau)$ were obtained for the transfer to the outdoor environment and for the transfer to the constant temperature layer respectively.

The use of value $R_\tau = 360$ generates a design climate expressed by mean values for 15 days of the outdoor temperature; this design climate is formed of the multi-annual means for each locality. In order to verify this simplifying hypothesis, the thermal response was analyzed in the form of the heat flow-rate density on the floor, for the case of thermal excitation caused by the variation of the actual outdoor temperatures, compared to the heat flow-rate specific to the thermal excitation represented by the multi-annual *monthly mean temperature*, as a climatic parameter variable in time.

For instance, during 312 consecutive hours in March 2003, the thermal flow, a result of the outdoor temperature variation was determined and compared to the value specific to March of the thermal response to the load generated by the sequence of the year days characterized by the multi-annual *monthly mean temperature*. For the case of the thermal excitation specific to the time lag in May 2003, the heat flow-rate density varies between 15 and 18 W / m^2, the mean being 16.5 W / m^2. In the case of the excitation based on multi-annual monthly means for March, the mean value of the heat flow-rate dissipated to the outside is 15.6 W / m^2.

The heat flow-rate density values were determined by the INVAR soft using the real climate in the period 1-14.03.2003, with outdoor temperature values as well as the annual climate and modeling by similitude for L = 0.50 m. The mean value provided by the measurements (flow-meter strip in the 15-20 cm outline zone) was 16.1 W / m^2. The maximum deviation of 5.45 % between the calculated values is considered acceptable, the more so as the flow channel length of 0.50m makes the structure rather sensitive at outdoor temperature variations in rather short time-lags (10-15 days) and the accuracy of the calculation model is confirmed by the mean value provided by the experiment (error of 2.48 %). For longer flow channels, the deviation is reduced, so that the correlation $q_{ext} = f(L)$, determined based on similitude modeling may be used. For one year, function $q_{ext} = f$ (L) is presented in fig. 9.

In order to use this calculation model, each flow channel is associated to the corresponding virtual outdoor temperature. This includes the effect of all the material layers on the flow channel line and synthesizes the phase lag corresponding to this line. The virtual outdoor temperature invariance in terms of the indoor temperature allows the tracing of calculation nomograms expressing the variation of the dissipated heat flow according to the period of the year and to the flow channels length, particularized for the specific conditions of each locality. The use of these nomograms avoids the necessity of determining the virtual outdoor temperatures in usual cases, as it is possible to simply use a calculation method specific to the steady-state conditions. Fig. 9 presents such a nomogram reflecting the variation of the unitary heat flow-rate dissipated to the outside, according to the flow channels length, for a period of one year.

The curves presented are valid for the structure of most of the floors on ground. It is interesting to notice the function of damping and phase lagging of the outdoor temperature oscillation performed by the ground, emphasized by the virtual outdoor temperature variation according to the real outdoor temperature variation. The diagram in fig. 10 presents the variation of the virtual outdoor temperature in terms of the variation of the annual outdoor temperature and of the flow channels length.

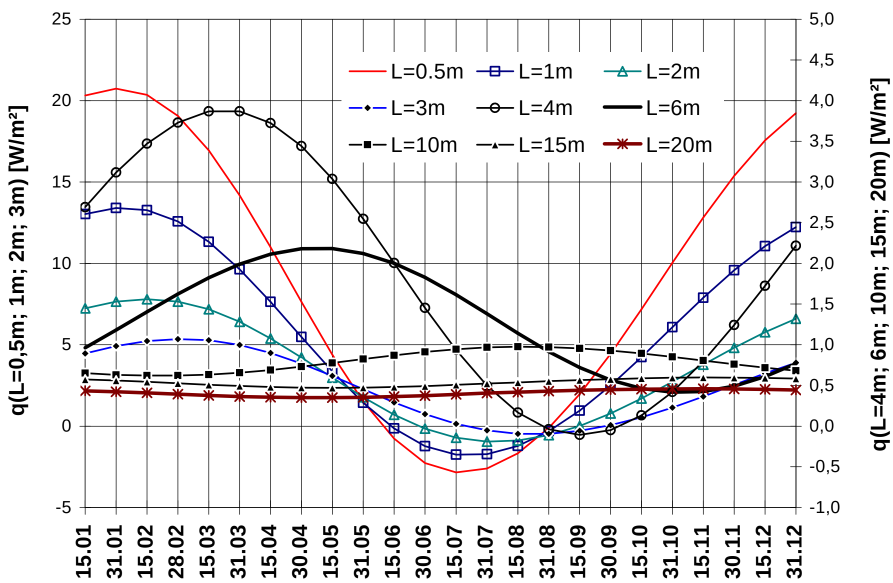

Fig. 9. Variation of the heat flow-rate dissipated to the outside through the floor on ground, according to the period of the year and to the flow channels length

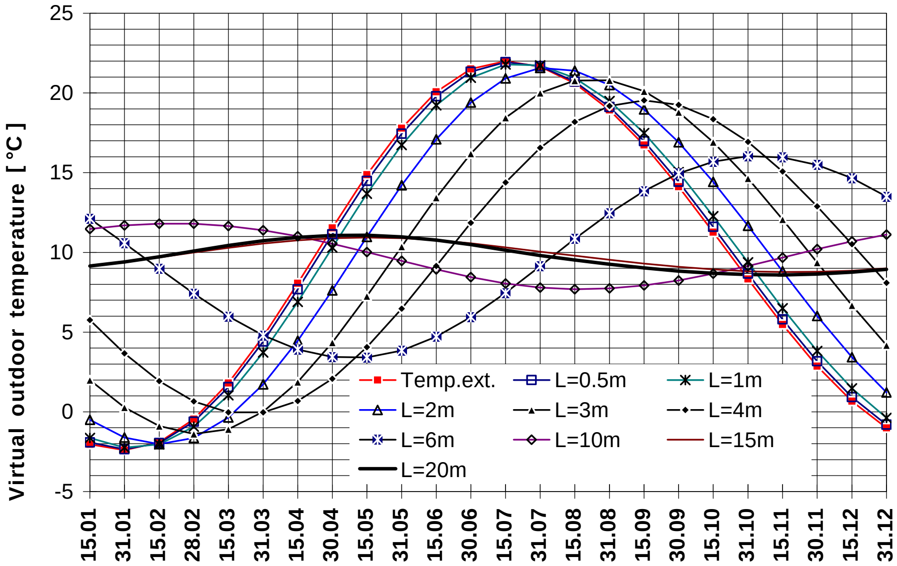

Fig. 10. Variation of the virtual outdoor temperature associated to the floor on ground – INCERC building

The relation used in determining the phase lag according to the flow channels length is:

$$D = 0{,}0295 \cdot L^4 - 0{,}8387 \cdot L^3 + \\ + 7{,}9482 \cdot L^2 - 6{,}989 \cdot L \qquad (43)$$

The relation used in determining the outdoor temperature oscillation damping according to the flow channels length is:

$$A = -9{,}36 \cdot 10^{-5} \cdot L^4 + 3{,}6104 \cdot 10^{-3} \cdot L^3 - \\ 4{,}1344 \cdot 10^{-2} \cdot L^2 + 6{,}4258 \cdot 10^{-2} \cdot L + 1 \qquad (44)$$

The heat transfer from the dwelled space to the groundwater uses the analysis model in transient conditions based on the similitude relations. The diagram in fig. 11 presents the variation of the ground under the floor in the significant days of the heating season. The aspect of the temperature curves supports the use of the transient heat transfer through ground. The specific heat flow-rate on the floor varies in time as in the diagram in fig. 12.

Based on the values of the specific heat flow-rate dissipated to the constant temperature

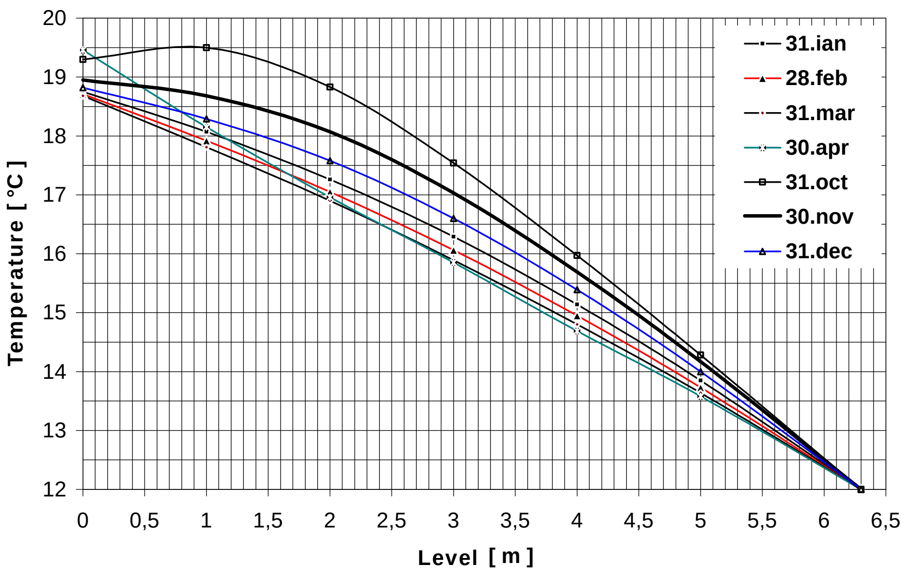

Fig. 11. Variation of the temperature in ground under the floor, towards the constant temperature layer – cold season

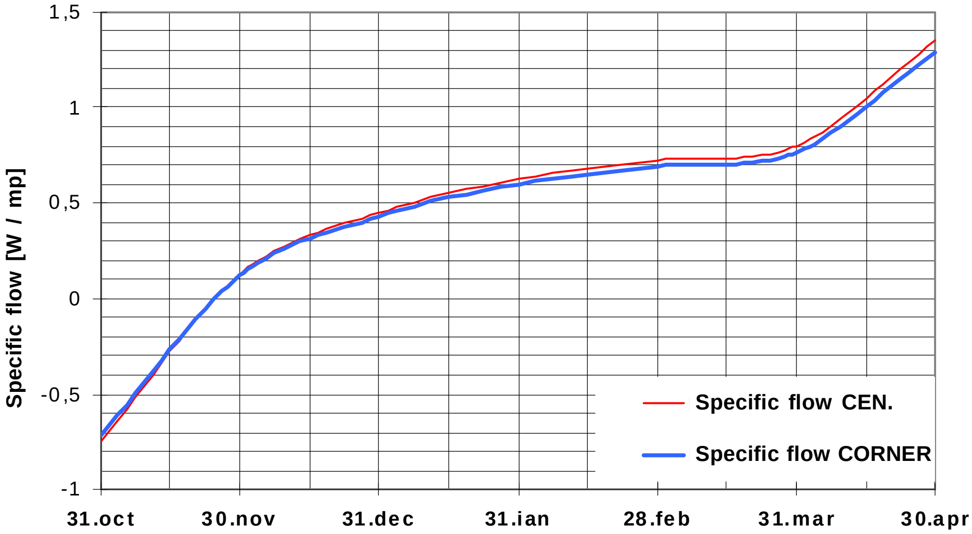

Fig. 12. Variation of the specific heat flow-rate on the floor towards the constant temperature layer – cold season

groundwater, the equivalent outdoor temperature $t_{ev_s}(\tau)$ is determined by relation:

$$t_{ev_s}(\tau) = t_i - R_s \cdot q_{Pd}(\tau) \tag{45}$$

where R_s is the thermal resistance between the room (enclosed space) and the final receiving layer.

The variation of $t_{ev_s}(\tau)$ is presented in the diagram in fig. 13.

5.3. Determination of the heat flow-rate on a building floor

Considering the elements described above, the heat flow-rate specific to the floor of a room (enclosed space) is determined based on its two components, the flow dissipated to the outside and the flow dissipated to the constant temperature layer.

Note: The term dissipation does not compulsorily mean the direction of the heat flow-rate to the two previously mentioned environments.

The total heat flow-rate on the entire floor is determined by the following relation:

$$Q(\tau) = \sum_j \frac{S_{jx}}{R_{jx}} \cdot [t_i - t_{ev_{jx}}(\tau)] + \sum_j \frac{S_{jy}}{R_{jy}} \cdot [t_i - t_{ev_{jy}}(\tau)] + \sum_j \frac{S_{jx}}{R_s} \cdot [t_i - t_{ev_s}(\tau)] \tag{46}$$

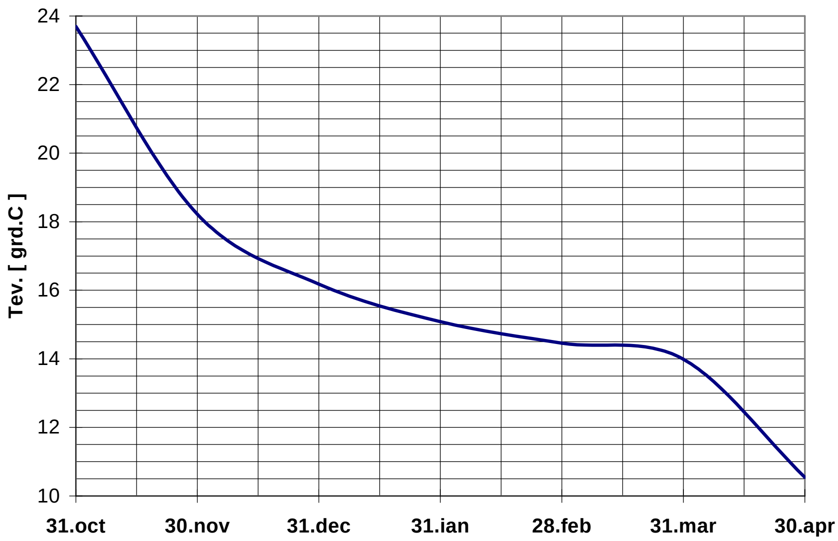

Fig. 13. Variation of the virtual outdoor temperature corresponding to the ground during the heat transfer to the groundwater

The following elements are defined:

$$\overline{R_e} = \left(\frac{1}{\overline{R_x}} + \frac{1}{\overline{R_y}} \right)^{-1} \quad (47)$$

$$\overline{R_x} = \frac{S_{LOC}}{\sum_j \frac{S_{jx}}{R_{jx}}}, \qquad \overline{R_y} = \frac{S_{LOC}}{\sum_j \frac{S_{jy}}{R_{jy}}} \quad (48)$$

$$t_{ev_e}(\tau) = \frac{\sum_j \frac{S_{jx}}{R_{jx}} \cdot t_{ev_{jx}}(\tau) + \sum_j \frac{S_{jy}}{R_{jy}} \cdot t_{ev_{jy}}(\tau)}{\frac{S_{LOC}}{\overline{R_e}}} \quad (49)$$

The heat flow-rate dissipated to the outside and to the groundwater through the floor of a room with the floor area S_{LOC} is determined by relation:

$$Q_{Pd}(\tau) = \frac{S_{LOC}}{\overline{R_e}} \cdot [t_i(\tau) - t_{ev_e}(\tau)] + \\ + \frac{S_{LOC}}{R_s} \cdot [t_i(\tau) - t_{ev_s}(\tau)] \quad (50)$$

The calculation based on the virtual outdoor temperature is not changed if $t_i = t_i(\tau)$. Values $t_{ev_{j\,x,\,y}}(\tau)$ do not change according to the modification of values $t_i(\tau)$. The values of the virtual outdoor temperatures is determined using fig. 10 ($t_{ev_e}(\tau)$) and fig. 13 ($t_{ev_s}(\tau)$). These values slowly vary so that constant values during one day may be considered in the practical calculations for one day.

6. PRACTICAL METHOD OF DETERMINING THE INTENSIVE (TEMPERATURES) AND EXTENSIVE (THERMAL FLOWS) THERMODYNAMIC PARAMETERS SPECIFIC TO THE HEAT TRANSFER BETWEEN THHE BUILDING AND GROUND

The fundamental calculation hypotheses are the following:

- The heat transfer is *transient* between the thermodynamic outline, represented by the area of the building adjoining the ground and the natural outdoor environment, represented by the outdoor air characterized by the virtual outdoor temperature specific to the flow channels connecting the two geometric marks previously mentioned as well as by the groundwater.

- The heat flow-rate generates flow channels and along them a heat transfer occurs between the indoor space (heated or not) and the natural outdoor environment. The configuration of the heat flow-rate lines is specific to the heat transfer steady-state conditions (arcs of circle). The heat transfer between two adjoining flow channels is overlooked.

6.1. Heat transfer characteristics (one-phase material)

From any point on the area representing the envelope in contact with the ground, a heat flow-rate is propagated to the groundwater, the temperature of which is t_a.

The heat flow-rate dissipated to the outside, from any point on the envelope, observes the principle of the "*least resistance path*".

The occupied and unoccupied spaces whose elements are located below the building perimeter Systematized Land Level (SLL) are characterized by a heat flow-rate dissipated to the outdoor natural environment or by a heat flow-rate received from the outdoor natural environment through the ground and the material layers forming the perimeter components lower than SLL.

The influence of certain spaces nearby, characterized by temperatures which are different from those of the natural outdoor environment, may be overlooked. The thermal balance relations use modified outdoor temperatures including the effects of phase-lagging and damping of the thermal waves specific to the extremely massive building components. (In this case the ground is assimilated to a building material).

In all the cases, the heat flow-rate generated by the heat transfer between the occupied or unoccupied spaces and the outdoor air are determined as well as the thermal flows generated by the existence of the groundwater in the ground.

The following cases are considered:

1. A space (occupied or unoccupied) characterized by temperature t_s (constant or variable according to the space thermal balance), confined by thermally not insulated vertical walls adjoining the ground, with a height h_s under SLL (systematized land level) as well as by a floor not thermally insulated;

2. A case similar to the previous one, but the vertical walls and the floor are thermally insulated;

3. Combinations between the situations specific to cases 1 and 2 with reference to the condition of the vertical walls and of the floor in terms of thermal insulation;

4. A building on a plinth with a height h_{sc} over SLL, with sub-cases:

4.1 Plinth and floor with no heat insulation;

4.2 Thermally insulated plinth and floor;

4.3 Combinations between the condition of the plinth and floor in terms of thermal insulation.

The relations for determining the thermal flow, in the case of *underground enclosures* have the following expressions:

$$Q_{e_k} = \frac{S_{lat} + S_{pard}}{\overline{R}_e} \cdot (t_s - \bar{t}_{eR_k}) \tag{51}$$

$$Q_{f_k} = \frac{S_{lat} + S_{pard}}{\overline{R}_f} \cdot (t_s - t_a) \tag{52}$$

and respectively:

$$Q_{sce_k} = \frac{S_{pard} + 4h_{sc}^2}{\overline{R}_{esc}} \cdot (t_s - \bar{t}_{esc_k}) \tag{53}$$

$$Q_{fsc_k} = \frac{S_{pard}}{R_{fsc}} \cdot (t_s - t_a) \tag{54}$$

in the case of the building *on an earth plinth*.

The temperatures of spaces, t_s, may be known based on the thermal and physiological comfort conditions and in this case:

$$t_s = t_{i0};$$

where t_{i0} is the design conventional indoor temperature, according to the type of room (enclosed space) or varies according to the spaces thermal balance, and then:

$$t_s = t_{sk}$$

The unoccupied spaces temperatures vary according to the variation of the outdoor climatic parameters and to the thermal flows specific to the equipments as well as to the building components adjoining the unoccupied spaces. The following cases are noticeable:

6.1.1. The unheated basement entirely occupies the space under the floor of the occupied spaces

The thermal balance equation, which is a linear algebraic equation with an unknown value, temperature t_{sk}, is the following:

$$\frac{S_{PL}}{R_{PL}} \cdot (t_{i0} - t_{sk}) + 2\pi A\delta_a \cdot (t_{apa} - t_{sk}) - Q_{e_k} - Q_{f_k} -$$
$$- 0{,}33 n_{asb} V_{sb} \cdot (t_{sk} - t_{ek}) - \frac{S_{Pesb}}{R_{Pesb}} \cdot (t_{sk} - t_{ek}) = 0 \tag{55}$$

where the thermal flows Q_{ek} and Q_{fk} are detailed in the form of relations (51) and (52).

6.1.2. The unheated basement partially occupies the space under the floor of the ground floor

Equation (55) is used, where S_{PL} is modified by adding the area adjoining an occupied space which is partially under the SLL level. The heat transfer through the floor of this space to the unoccupied basement is overlooked.

6.1.3. The basement is heated at temperature t_{i0}

In this case the heat flow-rate dissipated to the natural outdoor environment is determined by relations (51) and (52), where $t_s = t_{i0}$.

6.1.4. The thermal flows dissipated form the occupied space of a building on ground at level h_{sc} above SLL

The thermal flows dissipated form the ocuupied space of a building in ground at levele h_{sc} above SLL are determined by relations (53) and (54). Value $h_{sc} \leq 0$.

We mention that in all cases:

– $Q > 0$ means dissipated thermal flow;
– $Q < 0$ means penetrated thermal flow.

If the unoccupied space neighbours a space the temperature of which is unknown, the temperatures of both spaces are determined by solving the thermal balance linear equation system specific to each space. A simplified solution is presented for the case of three categories of unoccupied spaces (for instance basement, staircase and attic).

1. The basement thermal balance equation is equation (55) which includes an additional item generated by the heat transfer between the basement and the unoccupied space adjoining the basement and the occupied spaces. The following equation results:

$$\frac{S_{PL}}{R_{PL}} \cdot (t_{i0} - t_{sk}) + \frac{S_{s_1}}{R_{s_1}} \cdot \left[E_1 t_{i0} + (E_2 - 1) \cdot t_{sk} + E_3\right] + \\ + 2\pi A \delta_a \cdot (t_{apa} - t_{sk}) - Q_{e_k} - Q_{f_k} - \\ - 0{,}33 n_{asb} V_{sb} \cdot (t_{sk} - t_{ek}) - \frac{S_{Pesb}}{R_{Pesb}} \cdot (t_{sk} - t_{ek}) = 0 \quad (56)$$

which is solved in terms of temperature t_{sk}.

2. The temperature of the unoccupied space *1* is determined by the following relation:

$$t_{1_k} = E_1 t_{i0} + E_2 t_{s_k} + E_3 \quad (57)$$

3. The temperature of the second unoccupied space, *2*, is determined by the following relation:

$$t_{2_k} = B_2 t_{i0} + B_3 t_{s_k} + B_4 \quad (58)$$

Coefficients *B* and *E* are determined according to the geometric characteristics and to the thermal coupling coefficients specific to the areas separating the environments in thermal contact.

7. CONCLUSIONS

1. This article aims to present a calculation model necessary and proper to the assessment of the heat flow-rate dissipated at the boundary between the building and the ground.

2. The mathematical model conceived is a hybrid model combining in its structure both the operational advantages of the steady-state conditions of heat transfer through semi-finite environments (ground) with local perturbations (building) and the characteristics of the heat transfer by conduction in massive environments, with reference at the damping and phase-lagging of the thermal waves, both specific to the heat transfer in transient conditions.

3. The use of the outdoor environment virtual outdoor temperature as a climatic parameter variable in space and time allows the use of the calculation model in the case of the transfer to the outdoor environment, characterized by the air outdoor temperature as well as to the groundwater, characterized by the water temperature. The virtual outdoor temperature is determined by considering a number of concentric flow channels developed on the flow lines assimilated to concentric circles, which approximate, with no significant errors, the heat flow-rate curves perpendicular on the isothermal curves, a result of using the conforming transformations at the level of the building – ground outline. The value of the virtual outdoor temperature is determined following the use of the RTU model for a flat plate defined by the length of the flow lines on the path between the building boundary and the outdoor environment. The boundary conditions are of the third rank at the ground-outdoor environment boundary and of the fourth rank at the contact between the successive ground layers, on the mentioned path;

4. The integration of the conduction parabolic equation was performed by using the INVAR software on the support of a virtual equivalent structure (one-layer or multi-layer) identified in terms of geometry and of the thermophysical parameters by using the similitude criteria Fo = *id* and Bi = *id*. between a natural environment and a virtual environment of the model type.

5. The verification of the essentially transient nature of the heat transfer between the building and ground was based on the experiments performed in the cold seasons of the years 2003 and 2004 in the CE INCERC Bucharest experimental building. The results obtained in a measurement period of 312 consecutive hours in March 2003 are presented as well as the results of the modeling on a similar model and the results obtained by using the multi-annual monthly mean temperatures as outdoor loading. The error of 2.48 % between the experiment and the calculation for the previously mentioned period, associated to the deviation of 5.45 % generated by using the monthly mean temperatures as a function specific to the outdoor environment, certifies the adoption of the final calculation model based on the use of the monthly mean temperature as a representative function of the outdoor environment, regardless of the real climatic characteristics (which deviate from the multi-annual mean values), as well as of the similar virtual environment as a material support substituting the ground. The result, associated to the above mentioned experiment, is specific to a zone rather close to the building outline strip and is accurately confirmed by the measurements / modelings performed on the experimental building floor, in zones far from the outline zone.

6. Based on the calculation model, particular cases of configuration of the building – ground boundary were considered, which imply directly heated / unheated basements, equipped or not with systems flowing hot fluids, as well as buildings on thermally insulated / not insulated over-ground plinths. The thermal correspondents between zones with different temperatures are considered, as secondary zones of the building, and their temperatures are determined by solving the linear systems of algebraic equations based on the independence of the virtual outdoor temperature values of the temperature of the building occupied / unoccupied spaces.

7. A lucrative practical method was conceived based on the mathematical model presented; this method is currently included in the Romanian EPB (building energy performance) calculation method, Mc 001 / 1-2006.

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