FUNCTIONING PRINCIPLES AND HEAT ENGINEERING CALCULUS ELEMENTS SPECIFIC TO THE MC ACTIVE SOLAR HEATING SYSTEM

Dan CONSTANTINESCU

ABSTRACT

This paper introduces the functioning principle as well as dimensioning elements of a new active solar heating system using air as heat carrier. The air is heated in flat-plate collectors and then circulated through the building walls having circulation "hollows". The heat storage function is performed by a rockbed storage unit. The solar collectors have water-air double service and supply the warm water required by the building during the warm season as well. The simplified mathematical model of unsymmetrical heating in case of laminar flowing between parallel walls is also presented; it offers good results as compared to the accurate model.

Starting from the solutions of the simplified model, the thermal response of building units for five MC system solar heating functional schemes is analyzed. The analysis is accompanied by calculus examples proving the fact that the heating system under study is considerably better that active solar heating traditional solutions using static heating units supplies by heating carriers with rather high temperature values (50°C).

For Romania's climatic conditions, the same energy effects may be obtained using the new system, with collecting areas about twice smaller than those used in a traditional active solar heating system.

Keywords: solar systems, solar energy

REZUMAT

Acest articol prezintă principiul de funcționare precum si elemente de dimensionare ale unui nou sistem solar activ de încălzire care utilizează aerul ca fluid de lucru. Aerul este încălzit în captatoare solare plane si apoi este vehiculat prin pereții clădirii, prevăzuți în interior cu canale de aer. Stocajul căldurii este realizat într-un pat de rocă. Panourile solare au o funcționare duală, cu fluid de lucru aer sau apă, furnizând pe durata sezonului cald apa caldă necesară clădirii. Se prezintă de asemenea modelul matematic simplificat de calcul al încălzirii asimetrice pentru cazul curgerii laminare între pereții paraleli. Rezultatele obținute cu acest model de calcul sunt bune, comparativ cu cele obținute prin aplicarea unor modele mate-matice exacte. Pornind de la rezultatele obținute cu modelul simplificat de calcul, este analizat răspunsul termic al unității clădirii pentru cinci scheme funcționale de instalații solare. Analiza este însoțită de exemple de calcul care demonstrează că sistemul de încălzire studiat este considerabil mai bun decât soluțiile tradiționale de încălzire care utilizează unități de stocaj alimentate cu flude de lucru cu temperatură considerabil mai ridicată (50°C).

În cazul condițiilor climatice din România, aceleaşi efecte din punct de vedere energetic pot fi obținute de noul sistem prin utilizarea unei suprafețe de panouri solare cu o suprafață de aproximativ două ori mai mică decât cea utilizată de un sistem solar tradițional de încălzire utilizând energia solară.

Cuvinte cheie: instalații solare, energie solară

I. Introduction

The solar heating of dwelling and working spaces is commonly performed in two ways:

– passive heating;

– active heating.

The collecting, storing and heat consumption functions of passive system are carried out naturally by means usually include in building structure, and therefore the contribution of mechanical means in heating fluid transport is minimum, meant to improve the system performance and not conditioning its functioning.

The collecting, storing and heat consumption functions of active system are carried put by forced circulation – using mechanical means – of one or several heating fluids, conditioning the system functioning and the energy saving performance.

The solar passive heating systems generally supply the heat quantity required by spaces adjacent to the collecting area, the other rooms being heated by traditional systems. The system service is rather simple but it can't be controlled, the heat storage unit "discharge" random element representing a considerable disadvantage. The

*INCERC (National Building Research Institute), Bucharest, E-mail: dan.constantinescu@incerc2004.ro

active heating systems have autonomous service and theoretically may supply the dwelling space overall heat consumption.

Active heating systems with liquid (water, water and ethylene glycol, water and propylene glycol etc.) or gas (air) heat carrier are used at present, specially for experiments. Liquid heat carrier heating uses temperature values close to those of traditional heating units. The necessity of adopting these temperature levels results from the rather small areas required by dwelling space heating units. Heating with low temperature liquid heat carrier (floor heating, ceiling heating etc.) implies complicated technological problems. In the first case (static heating units or ventiloconvectors) the system is not advantageous as high service temperature values and consequently very sophisticated and expensive solar collectors are required. At the same time, the solar radiation collecting efficiency values are rather low ($\eta \cong 0.12$ for nonselective collectors and $\eta \cong 0.30$ for selective collectors[1]). The heat exchangers placed between the collecting loop and the storage circuit considerably diminish the system performance.

A cheap technical solution may be considered the one specific to active-passive systems, where active heating is used only for rooms placed on not-sunlit facades [1].

When air is used as heat carrier, part of the disadvantages caused by liquid heat carrier heating are avoided. The main advantage is the technical safety offered by the system (no anti-freeze solution is required). The low temperature levels required by this type of installations represents a further advantage with direct influence on the solar radiation collecting efficiency. This aspect should not be misunderstood by considering that the performance increase determined by the obviously low temperature values required by air-using systems is extremely spectacular and may be achieved without special collectors. F_R, the heat removal factor of the air collectors is lower as compared to the water collectors and consequently for the same heat loss overall coefficient, K_Σ, the air collector curve on its entire range will be situated under the water collector curve. The value of coefficient F_R depends on the nature of the heat transfer from the absorbing plate to the fluid and therefore air turbulent flow through the solar collectors is required. The immediate consequence is the increase of pressure drop and consequently and increased electrical energy consumption at the ventilators. Of course, correct designing will be the result of system energy consumption optimizing.

A disadvantage of warm air heating is the rather high steel consumption required by air circulation ducts. Furthermore, air circulation in closed circuit generates noxes spreading (smell, smoke, substances etc.) in the rooms.

In all cases described above, the building heat insulation should ensure low heat consumption so that the energy recovery obtained by the solar installations might be taken into consideration.

A general characteristic of the active systems described above is the fact that they represent adaptations of building traditional service (in terms of heat consumption) to the use of solar energy as heat source. From this viewpoint the passive system partially conform the building to the requirements of solar radiation direct collecting.

Of course, in case of active heating the solar collectors included in the facades represent the first step of functional synthesis, but the dwelling space basically works independently of the solar installation; the “part” of the dwelling space is that of exergy dissipation and that of the solar installation is exergy supplying[2]. This paper introduces a new system of dwelling space active heating using air.

II. Basic principles of MC system

a. The dwelling space heating heat carrier is circulated in a closed circuit through inside and outside building units (hollow walls).

[1] Under Romania’s cold season climatic conditions.

[2] It is important to point out that formulations like “energy consumption” are not correct an energy is not consumed in insulated systems. Nevertheless, in order to employ elements already used by engineers, this paper will use such expressions.

b. The windows have the function of elements recovering heat from the exhaust waste air.

c. The air circuit in building units may be use in summer as dwelling space cooling circuit.

d. The fresh air required by physiological comfort is preheated by the heat recovered from the air circulated through the external walls and from the exhaust waster air.

By using the building walls as static heating units with air as heat carrier, large heating (or cooling) areas are ensured and consequently the heat carrier temperature will be low. The effect of radiation to the walls is diminished as compared to warm air traditional heating, where the wall surface temperature is lower (for the same heat insulation rate). The surface temperature of the tight frame windows is therefore superior to the traditional systems. The air circulated back through the outside walls replaces the heat insulation (within limits that will be determined in this paper).

The dwelling space may be entirely or partially heated by this system, the remaining heat quantity being supplied by a traditional source.

Air heating is performed either by traditional means (MC-1) or by using a solar installation (MC-2). In case of MC-1 system, waste water, flue gases and other secondary heat sources may be used as air hating agents. As to the energy consumption required by ventilators, it becomes higher because of the air circulation in the hollow inside the building units. Still, a careful design may avoid this rather important disadvantage. Noxes spreading in the dwelling space is obviously avoided.

The main characteristic of MC type systems (especially MC-2) is the fact that the air circulated through the building units is basically used as heat insulating element and additionally as dwelling space heating agent. Thus a rather large heat quantity with low heat potential is used instead of a small heat quantity with rather high heat potential. Therefore solar energy actually contributes to heat saving, working as a "thermal barrier" against the heat losses from the heated space. In terms of solar collector service temperature, the advantage of the system is obvious: the temperature of the air circulated through the external walls is the lowest temperature of the entire heating system and therefore the collecting efficiency correlated with this temperature value will be the highest possible. As a parenthesis, we may consider that if the external air around a building is solarly heated from temperature t_e to temperature t_i of the air in the heated space, this building will not lose heat and therefore heat saving will be total. System MC-2 suggested by the author is a transitional stage between the traditional inside heated house and the adiabatic house without heat exchange with the environment.

The fact is worth mentioning that *a* (*n*) MC-2 solar building will have rather few freedom degrees from the architectural viewpoint. Of course, this may be an impediment in developing towns or villages according to system MC-2. The system may nevertheless be efficient for groups of buildings. It is important to point out that the solar radiation collecting units are of water-air combined type. They use air in the heating season and water in the warm season. Thus the neighbour consumers may be supplied in summer, the heat surplus contributing to the solar building overall economy. The system is therefore suited for solar and unsolar groups of buildings. [3]

III. Calculus elements

The paper will further introduce the method of determining the thermal response of the MC heating system constituent elements which are the following:

a. External wall

b. Partitions (inside walls)

c. Double glazed window adapted to heat recovery from the exhaust waste air.

In all cases the air circulated through the building units is started by mechanical means, therefore the flowing phenomenon is specific to forced flow with heat exchange in uncircular section ducts. Thanks to the requirement of limiting the load losses caused by air circulation on the one hand and by the necessity of not exceeding the velocity values producing noise in the installation on the other hand, flowing belongs to the laminar zone.

The mathematical modelling of the laminar flow between flat plates with heat exchange is rather complicated and in any case the final solutions are too sophisticated to be commonly used by building and installation designers. This paper suggests therefore a simplified mathematical model which, by the solutions suggested, facilitates the designer activity. The model proved valid by comparing the results offered by the analytical solution of the model of laminar flow between flat plates.

III.1. Heat transfer in laminar flow between parallel flat plates (unsymmetrical heat transfer)

Calculus hypotheses:

1. The heat transfer is steady-state. This hypothesis is supported by the fact that the system works permanently and the building unit thermal capacity may therefore be ignored.

2. Flowing is plane, the thermal effects being considered on two directions: along the air current and on the flowing section; axial conductibility in the air current is ignored.

3. A rodlike velocity profile is assumed. This hypothesis is actually applied by placing movement disturbers into the air flowing spaces; these disturbers will make the flow uniform and prevent preferential directions.

4. Flowing is considered stabilized from the thermal and hydraulic viewpoint, as the flowing lines are long ($L \geq 5$ m).

5. The physical properties of air are not changed according to temperature. This hypothesis is proved valid by the fact that temperature variation on the air flowing lines is rather small (< 10°C).

6. The overall and radiation heat exchange coefficient do not change according to air temperature. The justification of this hypothesis is similar to the one described for hypothesis 5.

The energy equation is written therefore in Cartesian coordinates as follows:

$$w \cdot \frac{\partial \vartheta}{\partial x} - a \cdot \frac{\partial^2 \vartheta}{\partial y^2} = 0 \qquad (1)$$

An initial condition as well as boundary conditions are associated with this equation. The initial condition actually is a boundary condition itself and it means to admit that in section $x = 0$ the air inlet temperature is constant its value being ϑ_0.

$$\vartheta\,(0, y) = \vartheta_0 \qquad (2)$$

The boundary conditions for the two areas 1 and 2 marking the flowing line express the heat balance specific to these areas, which separate the environment from the warm air flowing line.

Taking into account the sign of the temperature derivate as against coordinate y and the fact that the heat flow is an always positive physical measure, the heat balance relations expressing the boundary conditions are written as follows:

$$-\lambda \cdot \left.\frac{\partial \vartheta}{\partial y}\right|_{y=0} + \alpha_r \cdot (\vartheta|_{y=0} - \vartheta|_{y=\delta}) = K_1 \cdot (t_i - \vartheta|_{y=0}) \qquad (3_1)$$

$$-\lambda \cdot \left.\frac{\partial \vartheta}{\partial y}\right|_{y=\delta} + \alpha_r \cdot (\vartheta|_{y=0} - \vartheta|_{y=\delta}) = K_2 \cdot (\vartheta|_{y=\delta} - t_e) \qquad (3_2)$$

The solution of equation (1) is function $\vartheta_1 = \vartheta_1\,(x, y)$ expressed by relation:

$$\vartheta_1(x, y) = \sum_k [A_k \cos(m_k y) + B_k \sin(m_k y)] \cdot \exp\left(-m_k^2 \cdot \frac{a}{w} \cdot x\right) \qquad (4)$$

The fact that for $x = \infty$, $\vartheta = 0$ is to be noticed and therefore solution (4) should be completed with the solution specific to the temperature steady-state condition. The final solution of equation (1) is the following:

$$\vartheta(x,y) = A + By + \sum_k [A_k \cos(m_k y) +$$

$$+ B_k \sin(m_k y)] \cdot \exp\left(-m_k^2 \cdot \frac{a}{w} \cdot x\right) \qquad (5)$$

m_k – eigenvalues
A_k, B_k – eigenfuctions

By applying the boundary conditions (3_1) and (3_2), the eigenvalues and the expressions of coefficients A and B are provided by relation (5).

$$2 \cdot \alpha_r \cdot \frac{\lambda}{\delta} \cdot Z_k - [\alpha_r \cdot (K_1 + K_2) + K_1 K_2] \cdot \sin Z_k +$$

$$+\left(\frac{\lambda}{\delta}\right)^2 \cdot Z_k^2 \cdot \sin Z_k - \frac{\lambda}{\delta} \cdot (K_1 + K_2 + 2 \cdot \alpha_r) \cdot$$

$$\cdot Z_k \cdot \cos Z_k = 0 \qquad (6)$$

where product $m_k\delta$ was noted by Z_k.

$$A = \frac{K_2 \cdot t_e + K_1 \cdot \left(1 + \dfrac{K_2}{\alpha_r + \lambda/\delta}\right) \cdot t_i}{K_2 + K_1 \cdot \left(1 + \dfrac{K_2}{\alpha_r + \lambda/\delta}\right)};$$

$$B = -\frac{1}{\delta} \cdot \frac{K_1 \cdot (t_i - A)}{\alpha_r + \lambda/\delta} \qquad (7)$$

After a few elementary calculations, expression (5) may be finally writted as follows:

$$\vartheta(x,y) = A + By + \sum_k B_k \cdot [\beta_k \cos(Z_k \bar{y}) + \sin(Z_k \bar{y})] \cdot$$

$$\cdot \exp\left(-Z_k^2 \cdot \frac{2}{P_e} \cdot \frac{x}{\delta}\right) \qquad (8)$$

where

$$\beta_k = \frac{\alpha_r \cdot \sin Z_k + (\lambda/\delta) \cdot Z_k}{\alpha_r \cdot (1 - \cos Z_k) + K_1}; \quad \bar{y} = \frac{y}{\delta} < 1$$

P_e – Peclet's adimensional number

The eigenfunctions B_k are determined considering polynom orthogonality:

$$\varphi_k = \beta_k \cdot \cos(Z_k \bar{y}) + \sin(Z_k \bar{y}), \quad \bar{y} \in [0,1]$$

In order to demonstrate the orthogonality of polynoms φ_k, the following equality should be demonstrated:

$$\int_0^1 \varphi_k(\bar{y}) \varphi_j(\bar{y})\, \mathrm{d}y = 0 \ \text{ for } k \neq j \qquad (9)$$

If the Sturm-Liouville problem is referred to, **relation (9) may be replaced by relation (9**$_1$**)**:

$$\frac{\mathrm{d}\varphi_k(\bar{y})}{\mathrm{d}\bar{y}} \cdot \varphi_j(\bar{y}) - \frac{\mathrm{d}\varphi_j(\bar{y})}{\mathrm{d}(\bar{y})} \cdot \varphi_k(\bar{y}) = 0 \quad (9_1)$$

which results considering the eigenvalue equation (6). According to condition (2), the eigenfunctions B_k are determined using relation:

$$B_k = \frac{\int_0^1 \psi(y) \cdot [\beta_k \cdot \cos(Z_k \bar{y}) + \sin(Z_k \bar{y})]\, \mathrm{d}\bar{y}}{\int_0^1 [\beta_k \cdot \cos(Z_k \bar{y}) + \sin(Z_k \bar{y})]^2 \mathrm{d}\bar{y}} \qquad (10)$$

where

$$\psi(\bar{y}) = \vartheta_0 - A - B \cdot \delta \cdot \bar{y}$$

Thus solution ϑ (x, y) is completely determined.

The thermal response of the heating element (external wall) or double glazed window consists in determining the air final temperature (average temperature in the final section of the flowing line) as well as the heat flow-rate delivered by the heating element.

According to expression (8) of function ϑ (x, y), the air final temperature (in section $x =$ $= L$) is determined using relation:

$$\overline{\vartheta}_{fL} = A + 0{,}5 \cdot \delta \cdot B +$$

$$+ \sum_k B_k \cdot \left(\beta_k \cdot \frac{\sin Z_k}{Z_k} + \frac{1 - \cos Z_k}{Z_k}\right) \cdot$$

$$\cdot \exp\left(-Z_k^2 \cdot \frac{2}{P_e} \cdot \frac{L}{\delta}\right) \qquad (11)$$

The heat flow-rate is determined using relation:

$$Q_{CL} = H \cdot \delta \cdot w \cdot \gamma_{cp} \cdot (\vartheta_0 - \overline{\vartheta}_{fL}) \qquad (12)$$

III.2. Simplified mathematical model (unsymmetrical heat transfer)

It is introduced the calculus scheme.

The heat balance of the infinitesimal element d*x* generates the differential equation:

$$\frac{d\vartheta}{dx}+\frac{\alpha\cdot H}{G\cdot c_p}\cdot(\vartheta-t_{p1})+\frac{\alpha\cdot H}{G\cdot c_p}\cdot(\vartheta-t_{p2})=0 \qquad (13)$$

In the section of air inlet when admitted in the flowing line, the temperature is constant $\vartheta\big|_{x=0}=\vartheta_0$.

The heat balance specific to areas 1 and 2 marking the air current is expressed by the following equations:

$$\alpha\cdot(\vartheta-t_{p1})+\alpha_r\cdot(t_{p1}-t_{p2})=K_1\cdot(t_{p1}-t_i) \qquad (14_1)$$

$$\alpha\cdot(\vartheta-t_{p2})+\alpha_r\cdot(t_{p1}-t_{p2})=K_2\cdot(t_{p2}-t_e) \qquad (14_2)$$

Solution (15) is obtained by explaining values t_{p1} and t_{p2} according to parameter ϑ:

$$\Theta=\Theta_0\cdot\exp\left(-\frac{a_1}{gc_p}\cdot\xi\right)-\frac{a_2}{a_1}\cdot\left[1-\exp\left(-\frac{a_1}{gc_p}\cdot\xi\right)\right]\cdot(t_i-t_e) \qquad (15)$$

where

$$\Theta=\vartheta-t_i;\ \Theta_0=\vartheta_0-t_i;\ \xi=\frac{x}{L}$$

$$a_1=\alpha\cdot\left[\left(1+\frac{\alpha_r}{\alpha+\alpha_r+K_2}\right)\cdot\frac{(\alpha+\alpha_r+K_2)\cdot K_1+\alpha_r K_2}{(\alpha+\alpha_r+K_1)\,(\alpha+\alpha_r+K_2)-\alpha_r^2}+\frac{K_2}{\alpha+\alpha_r+K_2}\right]$$

$$a_2=\alpha\cdot\left[\left(1+\frac{\alpha_r}{\alpha+\alpha_r+K_2}\right)\cdot\frac{\alpha_r K_2}{(\alpha+\alpha_r+K_1)\,(\alpha+\alpha_r+K_2)-\alpha_r^2}+\frac{K_2}{\alpha+\alpha_r+K_2}\right]$$

The air final temperature results from relation (15) for $\xi=1$ and the heat flow-rate is determined using relation:

$$Q_c=G\cdot\delta\cdot w\cdot\gamma\cdot c_p[\Theta_0-\Theta(1)] \qquad (16)$$

The solutions offered by the simplified model are proved valid by comparing the thermal response (final temperature and heat flow-rate) calculated by the accurate method (see Chapter III.1 – relations (11) and (12)) to the thermal response calculated by the simplified method (relations (15) and (16)). The calculations were performed for the value range $Re \geq 1.500$. A rather general calculus example (representing most common cases which may occur in practice) is further presented.

Calculus data:

K_1 = 2.90 W / m²K
K_2 = 0.58 W / m²K
α_r = 3.48 W / m²K
α = 2.32 W / m²K
λ / δ = 1.16 W / m²K
L = 4.00 m
H = 3.00 m
Re = 2.000
Pe = 1.440
ϑ_0 = 24°C
t_i = 18°C
t_e = 0°C

The surface heat exchange coefficient α is determined by relation

$$\alpha=\mathrm{Nu}\cdot\frac{\lambda}{\Delta}\approx 4.00\cdot\frac{\lambda}{\Delta}$$

where Δ is the characteristic length, here $\Delta = 2\delta$, value Nu is determined according to Pe or Ra if convection prevails.

The result is as follows:

a. The first three solutions of the eigenvalue equation (6) together with the corresponding values of eigenfunctions are written in Table 1.

Table 1

Z_k	B_k
1.4969314	7.6514722
5.0693346	– 0.0687396
6.7152939	0.8980271

The solutions of the accurate method result from relations (11) and (12):

$$\overline{\vartheta}_{fL} = 21.07\ °\text{C};\ Q_{CL} = 169.14\ \text{W}$$

b. The thermal response determined by the simplified method using relations (15) and (16) is expressed by the following values:

$$\overline{\vartheta}_f = 21.01°\text{C},\ Q_C = 172.82\ \text{W}$$

c. The deviations from the exact solution are:

$$\varepsilon_{\overline{\vartheta}_f} = \frac{\overline{\vartheta}_f - \overline{\vartheta}_{fL}}{\overline{\vartheta}_{fL}} = -0.28\ \%,$$

$$\varepsilon_{Q_C} = \frac{Q_C - Q_{CL}}{Q_{CL}} = 2.1\ \%$$

For the range mentioned above (Re ≥ 1.500) the deviations remain around the same values. The simplified method may therefore represent the calculus basic of MC system.

III.3. Analysis of MC solar heating system functioning

The analysis is performed according to a number of calculus schemes simulating various possible constructive solutions.

a.1. The first scheme refers to the heating of a space separated from the environment by hollow walls through which warm air is circulated. **The dwelling space temperature is** t_i, the environment temperature is t_e and the heat exchange areas of the walls are noted S_j, index j characterizing one wall. The ideal case of air circulating through the entire hollow wall is referred to.

For each surface S_j, the air final temperature is determined using the following relations:

$$\Theta_{S1} = \Theta_0 E_1 - \frac{a_2}{a_1}\cdot(t_i - t_e)\cdot(1 - E_1)$$

$$\Theta_{S2} = \Theta_{S1} E_2 - \frac{a_2}{a_1}\cdot(t_i - t_e)\cdot(1 - E_2)$$

...

$$\Theta_{Sj} = \Theta_{Sj-1} E_j - \frac{a_2}{a_1}\cdot(t_i - t_e)\cdot(1 - E_j)$$

The results is

$$\Theta_{Sj} = \left[\Theta_0 + \frac{a_2}{a_1}\cdot(t_i - t_e)\right]\cdot\prod_1^j E_j - \frac{a_2}{a_1}\cdot(t_i - t_e) \tag{17}$$

The heat flow-rate supplied by the heating source (SI) is determined by relation

$$Q_{SI} = G\cdot c_p\cdot\left[\Theta_0 + \frac{a_2}{a_1}\cdot(t_i - t_e)\right]\cdot(1 - \prod_1^j E_j) \tag{18}$$

In case of dwelling space traditional heating, the average hourly heat consumption, equal to the heat flow-rate of the heating source, is given by relation

$$Q_{SI}^{CL} = \sum_j S_j\cdot K\cdot(t_i - t_e) \tag{19}$$

In both relations (18) and (19) the walls are supposed to be identical as construction. The condition is raised that flow-rate Q_{SI} should represent a part "up" of flow-rate

$$Q_{SI}^{CL} = G\cdot c_p\cdot\left[\Theta_0 + \frac{a_2}{a_1}\cdot(t_i - t_e)\right]\cdot(1 - \prod_j E_j) =$$

$$= p\cdot\sum_j S_j\cdot K\cdot(t_i - t_e) \tag{20}$$

If everything is reduced to the heat exchange unitary area, equation (20) leads to solution

$$p = \frac{G \cdot c_p}{K \cdot \sum_j S_j} \cdot \left(\frac{\Theta_0}{t_i - t_e} + \frac{a_2}{a_1} \right) \cdot (1 - \prod_j E_j) \tag{21}$$

Chapter III.2 proves that expressions E_j have form:

$$E_j = \exp\left(-\frac{a_i}{g_i c_p} \right) \text{ where } g_i = \frac{G}{S_j}$$

Therefore

$$\prod_1^j E_j = \exp\left(-a_1 \cdot \frac{\sum_j S_j}{G \cdot c_p} \right) \tag{22}$$

or

$$\prod_1^j E_j = \exp\left(-\frac{a_1}{g c_p} \right) \tag{23}$$

where g is specific flow-rate of air circulated through the hollow walls.

The required replacements having been performed, relation (21) becomes

$$p = P \cdot \frac{a_1}{K} \cdot \left(\frac{\Theta_0}{t_i - t_e} + \frac{a_2}{a_1} \right) \tag{24}$$

where

$$P = \frac{g \cdot c_p}{a_1} \cdot \left[1 - \exp\left(-\frac{a_1}{g \cdot c_p} \right) \right]$$

The second condition required in defining the system functioning is that the inside temperature reached should be t_i. This condition is mathematically expressed by relation

$$\int_0^{L_j} H \cdot K_1 \cdot (t_{p1} - t_i)\, \mathrm{d}x = 0 \tag{25}$$

Relation (25) proves the fact that the sum of the heat flows specific to the heat exchange areas of the heated space is null; this condition is identical with the balance between heat supplies and heat losses. It clearly comes out that there will be parts where $\vartheta > t_i$ and parts where $\vartheta < t_i$. If $\vartheta > t_i$ on the entire line circulated by air, the inside temperature achieved will obviously be $t'_i > t_i$. Relation (25) therefore provides a correlation between the flow-rate of the circulated air and the inlet temperature of air when admitted in the circulation line through the hollow walls. Relation (25) becomes

$$P \cdot \left(\frac{\Theta_0}{t_i - t_e} + \frac{a_2}{a_1} \right) = \frac{a_2}{a_1} + \\ + \frac{\alpha_r \cdot K_2}{\alpha \cdot (2\alpha_r + K_2 + \alpha)} \tag{26}$$

Relation (27) is obtained by replacing (26) in equation (24):

$$p = \frac{a_2}{K} + \frac{a_1}{K} \cdot \frac{\alpha_r \cdot K_2}{\alpha \cdot (2\alpha_r + K_2 + \alpha)} \tag{27}$$

Relation (27) proves that the heat consumption ratio depends neither on the temperature admitted in the hollow wall circuit nor on the circulated air flow-rate. Nevertheless these values should not be considered unimportant for the system performance. They are correlated for a given system according to relation (26). The analysis of relations (26) and (27) according to the specific flow-rate of the air circulated through the walls and to coefficients K_1 and K_2 proves the fact that the heat consumption ratio p actually keeps a constant value $p \approx 1.03$ for all cases.

a.2. The second case subject to analysis is the endowment of the external walls with hollows in a subunitary rate m. The correlation between the temperature of air when admitted in the circuit and the air specific flow-rate is given by relation

$$P \cdot \left(\frac{\Theta_0}{t_i - t_e} + \frac{a_2}{a_1} \right) = \frac{a_2}{a_1} + \frac{\alpha_r K_2}{\alpha \cdot (2\alpha_r + K_2 + \alpha)} + \\ + \frac{1-m}{m} \cdot \frac{K}{K_1} \cdot \frac{(\alpha + \alpha_r + K_1) \cdot (\alpha + \alpha_r + K_2) - \alpha_r^2}{\alpha \cdot (2\alpha_r + K_2 + \alpha)} \tag{28}$$

and the heat consumption ratio in the new variant as against traditional heating is

$$p = m \cdot \left[\frac{a_2}{K} + \frac{a_1}{K} \cdot \frac{\alpha_r K_2}{\alpha \cdot (2\alpha_r + K_2 + \alpha)} \right] +$$

$$+ (1-m) \cdot \frac{a_1}{K_1} \cdot \frac{(\alpha + \alpha_r + K_1) \cdot (\alpha + \alpha_r + K_2) - \alpha_r^2}{\alpha \cdot (2\alpha_r + K_2 + \alpha)} \tag{29}$$

Relation (29) is represented as line $p = p(m)$ which is general.

The fact should be noticed that consumption in variant MC will increase with about 10 % as compared to dwelling space traditional heating systems using static heating units. The low temperature level of the heating air, $\overline{\vartheta} \approx 24°\text{C}$, should also be mentioned; this may lead to an efficient collecting of solar radiation. This aspect will be more dealt with further on. In both cases described above, the heating walls supply the entire heat consumption.

a.3. The third case subject to analysis is a room using, beside the heating walls, a traditional source (e.g. radiators with function of auxiliary source), in order to supply the heat consumption requirement.

The case $m = 1$ will analysis, where m has the function described above.

In order to reach inside the require temperature t_i, the auxiliary source Q_{AUX} should balance the heat flow lost through the walls towards the moving air space with temperature $\vartheta(x)$. This condition is mathematically expressed under the form of the following relation:

$$Q_{AUX} = K_1 \cdot H \cdot \int_0^L (t_i - t_{p1})\, \mathrm{d}x \tag{30}$$

which becomes

$$Q_{AUX} = S_T K_1 \cdot \frac{\alpha_r K_2 + (2\alpha_r + K_2 + \alpha) \cdot \frac{a_2}{a_1}}{(\alpha + \alpha_r + K_1) \cdot (\alpha + \alpha_r + K_2) - \alpha_r^2} \cdot$$

$$\cdot (t_i - t_e) + K_1 S_T \cdot \frac{\alpha \cdot (2\alpha_r + K_2 + \alpha)}{(\alpha + \alpha_r + K_1) \cdot (\alpha + \alpha_r + K_2) - \alpha_r^2} -$$

$$- P \cdot \left[\Theta_0 + \frac{a_2}{a_1} \cdot (t_i - t_e) \right] \tag{31}$$

The heat consumption required by the heating of the air circulated through the hollow walls is determined by relation:

$$Q_{SI} = S_T \cdot P \cdot \left[\Theta_0 + \frac{a_2}{a_1} \cdot (t_i - t_e) \right] \cdot a_1 \tag{32}$$

The overall heat consumption is therefore given by the following relation:

$$Q_T = Q_{AUX} + Q_{SI} = S_T A_1 \cdot (t_i - t_e) + A_2 Q_{SI} \tag{33}$$

where

$$A_1 = K_1 \cdot \frac{\alpha_r K_2 + \alpha \cdot (2\alpha_r + K_2 + \alpha) \cdot \frac{a_2}{a_1}}{(\alpha + \alpha_r + K_1) \cdot (\alpha + \alpha_r + K_2) - \alpha_r^2}$$

$$A_2 = 1 - \frac{\alpha \cdot (2\alpha_r + K_2 + \alpha)}{(\alpha + \alpha_r + K_1) \cdot (\alpha + \alpha_r + K_2) - \alpha_r^2} \cdot \frac{K_1}{a_1}$$

If the terms of relation (33) are divided by S_T the result is

$$q_T = q_{AUX} + q_{SI} = A_1 \cdot (t_i - t_e) + A_2 q_{SI}$$

Sum (q_{AUX} + q_{SI}) is noticed to have a minimum value for $q_{SI} = 0$ ($q_{SI} \geq 0$ is stated for all cases). This value corresponds to the heat consumption of the room under study, whose external walls are doubled inside. In this particular case, the overall heat exchange coefficient is lower than the one specific to external walls which are not doubled inside. This effect is actually obtained by recirculating – without heating – the air in the ventilated space.

At the same time it is worth mentioning that $q_{SI} = 0$ leads to a maximum q_{AUX}, which represents a disadvantage for solar heating, the auxiliary source being supplied by a heat carrier with rather high temperature (about 50°C). As a consequence, even if the room overall heat consumption decreases together with value q_{SI}, this solution is disadvantageous in terms of solar energy use. For an actual case requiring a constructive solution, functions $q_T = f_1(q_{SI})$, $q_{AUX} = f_2(q_{SI})$, $\Theta_0 = f_3(q_{SI})$ and $SP = f_4(q_{SI})$ are represented, where *SP* is the specific solar radiation collecting area (referred to the area of the external walls), the house using only solar energy for heating. The collecting area necessary to supply q_{AUX} and q_{SI} were determined according to the characteristic curves of the Romanian solar collectors which are extensively produced at present, to the differentiat temperature level required by the two heat sources and to the climat specific to the heating season in Romania. A clear difference between the corresponding values $q_{SI} = 0$, $SP = 0.55$ and $q_{AUX} = 0$, $SP = 0.28$ is to be noticed. Although q_T increases from value 7.83 W / m^2 in the first case to 9.86 W / m^2 in the second case, the collecting area necessary for heat consumption entirely covering is reduced to a half. This is the direct consequence of the low service temperature level required by the solar installation in variant MC.

b.1. The fourth case is a room using the inside walls as heating elements circulated by the warm air, which returns into the heat storage unit through the external walls.

The room heat balance is described by relation

$$Q_{PI} + Q_{AUX} = Q_{PE} \qquad (34)$$

where

$$Q_{PE} = K_1 S_T \int_0^1 (t_i - t_{p1})\, d\xi$$

In this case value $Q_{AUX} \geq 0$, the same as in the case previously described.

The air temperature variation on the interval wall line is given by relation

$$\Theta = \Theta_0 \cdot \exp\left(-\frac{1}{g \cdot c_p} \cdot \frac{S_I}{S_T} \cdot \frac{\alpha K_1}{\alpha + K_1} \cdot \xi \right) \qquad (35)$$

The result is:

$$Q_{PI} = g \cdot c_p \cdot S_T \cdot \Theta_0 \cdot \left[1 - \exp\left(-\frac{1}{g \cdot c_p} \cdot \frac{S_I}{S_T} \cdot \frac{\alpha K_1}{\alpha + K_1} \right) \right] \qquad (36)$$

The heat consumption required by the air circulating through the external walls is determined by relation (32) where Θ_0 is replaced by value

$$\Theta_0' = \Theta_0 \cdot \exp\left(-\frac{1}{g \cdot c_p} \cdot \frac{S_I}{S_T} \cdot \frac{\alpha K_1}{\alpha + K_1} \right) \qquad (37)$$

Therefore

$$Q_{SI}^1 = S_T \cdot P \cdot \left[\Theta_0 \cdot f(S_I / S_T) + \frac{a_2}{a_1} \cdot (t_i - t_e) \right] \cdot a_1 \qquad (38)$$

The heat loss through the external walls Q_{PE} is determined using relation (31) changed as follows:

$$Q_{PE} = S_T \cdot A_1 \cdot (t_i - t_e) - (1 - A_2) \cdot Q_{SI}^1 \qquad (39)$$

Thus relation (34) becomes an equation where the unknown factor is Q_{AUX}:

$$Q_{AUX} = Q_{PE} - Q_{PI}$$

Considering expressions (36) and (39) of Q_{PI} and Q_{PE} respectively and reducing everything to the external wall unitary area, relation (40) is obtained:

$$q_{AUX} = \varphi(\Theta, S_I / S_T) \qquad (40)$$

In order to observe condition $q_{AUX} \geq 0$, equation (40) provides the boundary correlation between values Θ_0 and $S_I / S_T : 0 \leq \Theta_0 \leq \Psi(S_I / S_T)$.

Using the calculus data of the previous example, the solutions of the problem are presented for two cases: $S_I / S_T = 1$ and $S_I / S_T = 7$.

In order to observe condition $q_{AUX} \geq 0$, equation (40) provides the value ranges for Θ_0:

a. For average winter conditions and $S_I / S_T = 1$.

$$0 \leq \Theta_0 \leq 1.30$$

b. For average winter conditions and $S_I / S_T = 7$.

$$0 \leq \Theta_0 \leq 0.68$$

The result is that for $S_I / S_T = 1$ and $q_{AUX} = 0$, value $SP = 0.26$ m² / m² and for $S_I / S_T = 7$ and $q_{AUX} = 0$, value $SP = 0.24$ m² / m².

The following result is obtained by comparing the results provided by case a.3. with those specific to case b.1., according to Θ_0 values.

$\Theta_0 = 0.68$

case a.3. $SP = 0.33$ m² / m²
case b.1. ($S_I / S_T = 7$) $SP = 0.24$ m² / m²

$\Theta_0 = 1.30$

case a.3. $SP = 0.29$ m² / m²
case b.1. ($S_I / S_T = 1$) $SP = 0.26$ m² / m²

The geometric locus of the $q_{AUX} = 0$ points offers an information very useful for design calculus: the influence rather reduced. In the case under study value $SP = 0.25$ m² / m² associated with value $\Theta_0 = 1$°C may be considered for any $S_I / S_T > 1$, for average winter conditions.

The analyses performed on cases a.3. and b.1. refer situation $m = 1$. Such situation do not usually occur in common design activity, as the actual value m are subunitary.

c.1. The fifth case under analysis in this paper is represented by the double glazed windows with function of heat recovery from the exhaust air.

In terms of mathematical modelling this case represents a particular situation of case a.1. Air temperature variation along the air flowing line is given by relation (15) where $\Theta_0 = 0$, therefore

$$\Theta = -\frac{a_2}{a_1} \cdot (t_i - t_e) \cdot \left[1 - \exp\left(-\frac{a_1 \cdot x}{g \cdot c_p} \right) \right] \quad (41)$$

The heat flow lost by the window in variant MC by transmission is determined using relation

$$q_T = K_1 \cdot \int_0^1 (t_i - tg_1) \, \mathrm{d}x \quad (42)$$

where tg_1 is temperature of inside glazing external area.

If tg_1 is explained according to Θ and taking into account (41), relation (42) becomes

$$q_T = K_1 \cdot \left[\frac{\alpha \cdot (2\alpha_r + K_2 + \alpha)}{(\alpha + \alpha_r + K_1) \cdot (\alpha + \alpha_r + K_2) - \alpha_r^2} \cdot (1 - P) + \right.$$
$$\left. + \frac{\alpha_r \cdot K_2}{(\alpha + \alpha_r + K_1) \cdot (\alpha + \alpha_r + K_2) - \alpha_r^2} \right] \cdot (t_i - t_e) \quad (43)$$

If the calculus example is referred to, the following relations are obtained for a double glazed window:

$$q_T = 1.43 \cdot (t_i - t_e) \text{ W / m}^2$$

as against the classical situation

$$q_T^{CL} = 2.06 \cdot (t_i - t_e) \text{ W / m}^2$$

The heat recovering window leads therefore to a 30 % heat consumption reduction due to transmission. The heat consumption required by fresh air heating will not be change.

If a heat recovery unit is placed at the window air outlet, the heat consumption required by fresh air heating will decrease. If for instance the heat recovery unit efficiency is 70 %, for the previously described calculus example, the heat consumption required by fresh air heating will be reduced by 50 %.

The possibility of performing building with an overall heat consumption reduced by about 15 % only by using MC-1 system as well as autonomous energy supplying systems by using MC-2 system, is proved valid by all cases described in this paper.

IV. Conclusions

This paper introduces the functioning and dimensioning principles of an active solar heating system, called MC by the author. This system uses low temperature warm air ($\vartheta_0 = 22 \div 24$°C) for dwelling space heating; this air is circulated through the partitions and external walls of the building. The air is heated by the water-air solar collectors (double service collectors) and admitted in the hollow walls by ventilators. Considering the rather important pressure drop on the air lines, a laminar flow through the walls is required. This paper introduces a mathematical model specific to nonisothermal laminar flow between parallel flat plates. This model proves valid a simplified mathematical model based on the overall heat balance. The simplified mathematical model offers solutions with deviations of about 0.3 % for temperature and of about 2 % for the heat flow, as compared to the model considered exact.

The calculus relations specific to MC heating system are determined according to the simplified model. Five cases of constructive solutions are studied as follows:

a.1. – the warm air is circulated through the external walls of the building, the "hollow" coefficient in the wall mass being $m = 1$;

a.2. – the air is circulated through the hollow partitions, the hollow coefficient **being** $m \leq 1$;

a.3. – the warm air is admitted through the building external walls and heating is performed together with a traditional auxiliary source supplied by a rather high temperature heat carrier (from solar heating viewpoint $\vartheta \cong 50°C$);

b.1. – the warm air is admitted through building partitions and returns to the heat storage unit through the circulation lines inside the external walls. Room heating is performed by the complementary service of a traditional heating source;

c.1. – exhaust air is exhausted through the interspaces between the double window panes.

In case a.1., a.2., a.3., b.1., heat consumption by transmission in variant MC exceeds with maximum 7 % the heat consumption of the same house without warm air circulation through walls. This increase is generated by a low temperature heat carrier whose temperature may be ensured easily by solar radiation collecting.

In case c.1., the heat consumption required by heat transmission through window decreases with about 30 % and the heat consumption required by fresh air heating decreased with about 50 % if air-air heat recovery units are used.

The analysis is accompanied by calculus examples proving the fact that by using MC solar hating system, the solar radiation collecting areas are smaller than those required by conventional active heating systems (with water heat carrier of about 50°C), with the same energy supplying effects.

SYMBOLS

ϑ_0 – warm air initial temperature [°C]

$\overline{\vartheta}_f$ – warm air final temperature [°C]

t_i – inside temperature [°C]

t_e – outside temperature [°C]

t_{p1} – temperature of partition inside area [°C]

t_{p2} – temperature of external wall inside area [°C]

K_1 – overall heat exchange coefficient from partition inside are towards inside air [W / m²K]

K_2 – overall heat exchange coefficient from external inside area towards aoutside air [W / m²K]

α_r – radiation heat exchange coefficient between partition inside areas and external wall [W / m²K]

α – convection heat exchange coefficient [W / m²K]

K – overall heat exchange coefficient of external walls not arranged for warm air circulation [W / m²K]

δ – thickness of air circulation space inside walls [m]

H – thickness of heating walls [m]

L – length of air flowing line into heating walls [m]

S_T – area of external walls with air circulation [m²]

S_I – area of partitions with air circulation [m²]

S_P – solar radiation collecting area [m²]

SP – solar radiation specific collecting area (S_p / S_T) [m² / m²]

Q_{AUX} – auxiliary source heat flow-rate [W]

Q_{PE} – heat flow-rate yielded towards external walls [W]

Q_{PI} – heat flow-rate yielded by heating partitions [W]

Q^1_{SI} – heat flow-rate required by heating of air circulating through hollow external walls [W]

q_{AUX} – auxiliary source heat flow-rate (Q_{AUX} / S_T) [W / m²]

q_{SI} – heat flow-rate required by heating of air circulating through walls (Q_{SI} / S_T) [W / m²] *a* – air thermal diffusivity [m^2 / s]

l – air thermal conductivity [W / mK]

c_p – air specific heat at constant pressure [J / kgK]

g – air specific weight [N / m^3]

w – air circulation velocity in heating walls [m / s]

G – heating air flow-rate [kg / s]

g – heating air specific flow-rate (G / S_T) [kg / m^2s]

Nu – Nusslet's dimensionless number

Pe – Peclet's dimensionless number

Ra – Raylayght's dimensionless number

m – rate of hollows in heating walls

p – ratio of heat consumptions of a house hated in system MC and in traditional system

η_{AUX} – solar radiation collecting efficiency at the temperature level required by auxiliary source

η_{SI} – solar radiation collecting efficiency at the temperature level required by MC system

m_K – eigenvalues

A_K, B_K – eigenfunctions

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