ANALYSIS MODEL FOR SOLAR INSTALLATIONS WITH SHORT-TIME STORAGE AND LIQUID STORAGE MEDIUM

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ABSTRACT

The heat storage unit is an important factor in determining the technical performance specific to a solar installation used in warm water producing on in space heating. The practical solutions used at present are economically based on short-time storage unit. Most installations have water sensible heat storage. For most small and mean installations, the calculus is based on two operational hypotheses of the heat storage unit: with perfect temperature stratification and with water uniform temperature. Both calculus models are limit cases, so they are not realistic. This paper introduces a calculus method based on the thermal response of the heat storage unit at impulsional excitations. Starting from the heat storage unit thermal response, the installation overall performance for long periods of time may be studied. This method proves valid for large central installations ($S_p \geq$ 2,00 m^2) and requires computer calculus. The solution described in the paper is part of an INCERC research computer program.

Key words: solar systems, thermal storage, renewable energy

REZUMAT

Unitatea de stocaj termic este un element cu rol important în performanța tehnică specifică a unei instalații solare utilizată pentru producerea de apă caldă menajeră sau pentru încălzirea spațiilor. Soluțiile practice din punct de vedere economic utilizate în prezent sunt de tipul unităților de stocaj pe intervale scurte de timp. Majoritatea instalațiilor au unități de stocaj în regim sensibil, utilizând ca agent de lucru apa. Pentru majoritatea instalațiilor de dimensiuni mici şi medii, calculul se bazează pe două ipoteze de funcționare a unității de stocaj termic: cu stratificare perfectă a apei în funcție de temperatură, respectiv cu temperatură uniformă a mediului de stocaj. Ambele modele de calcul sunt cazuri extreme, deci nu sunt realiste. Prezentul articol introduce o metodă de calcul bazată pe răspunsul termic al unității de stocaj termic la excitații de tip impulsional. Pornind de la răspunsul unității de stocaj termic, poate fi studiată performanța globală a instalației pe intervale lungi de timp. Această metodă este validată pe instalații *centrale* de mari dimensiuni ($S_p \geq 2{,}00$ m^2) şi necesită calcul computerizat. Soluția descrisă în prezentul articol este parte a unui program de cercetare INCERC.

Cuvinte cheie: instalații solare, stocaj termic, energie regenerabilă

I. Introduction

Common warm water producing solar installations (used either for dwelling space heating in active heating installations or to supply domestic or technological requirements) generally include heat storage units. Such installations are presently equipped with short-time storage units (STSU), because of their high price which imposes economic restrictions. The heat storage unit is considerably important for the installation energy balance: solar radiation collecting efficiency depends on storage unit temperature level. Two simplified calculus models are used in solar installation dimension-ing practical calculus. One of them [1] implies a perfect stratification of the STSU water temperature, so that during the heat accumulation period, the temperature value of the water introduced in the collecting loop is constant and equal to the cold water temperature value. The other model [2] implies a uniform temperature for each moment in STSU mass. This model also has an improved variant with the analysis performed on uniform temperature sections, using three calculus nodes [2]. Both models are particular cases of solar installation service:

– the stratification model corresponds to heat carrier low flow-rate values (the STSU water is circulated once a day through the solar collectors or through the solar installation heat exchangers);

– the uniform temperature model corresponds to heat carrier high flow-rate

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values (the STSU mass water is recalculated within less than an hour through the solar collectors or through the solar installation heat exchangers).

Any complete analysis of solar installations considered in calculus different heat carrier flow-rate values, without using different mathematical models. The heat carrier flow-rate represents a decision taken according to the analysis of the effect on the installation overall performance.

This paper introduces an analysis method of solar installation service based on STSU thermal response. The STSU heat balance is described by the mathematical model of the unisothermal flow through cylindrical pipes with uniform velocity profile.

The first part describes the analytical solution of the problem, using impulsional functions as thermal excitation function.

The second part briefly describes the way of using the method in the system overall analysis required by solar installation dimensioning. We have to mention that the analytical solution is valid only for large seasonal domestic warm water producing installations ($S_p \geq 2{,}00$ m^2) with the cold water temperature value close to the environment temperature or for heating installations with storage tanks placed in heated spaces ($t_s > t_{ext}$).

II. STSU unitary thermal response

For practical reasons, the STSU will be cylindrical and the heat carrier will flow along the generatrix. As the section of commonly used storage tanks is rather large, the heat carrier velocity is considered constant and with uniform profile. The thermophysical properties of the heat carrier do not vary according to temperature.

The equation describing the STSU thermal behaviour is the Khirchoff-Fourier equation for transient conditions, written within cylindrical coordinates and without heat sources (fig. 1). This equation may be integrated only if the initial and boundary conditions are stated.

As the SYSU has limited functionality in time (generally ≅ 24 h), the initial temperature may be considered to coincide with the lowest temperature in the system (e.g. for warm water producing installations, with the cold water temperature). As concerns the boundary conditions, they describe the heat losses from the storage tank water to environment, through

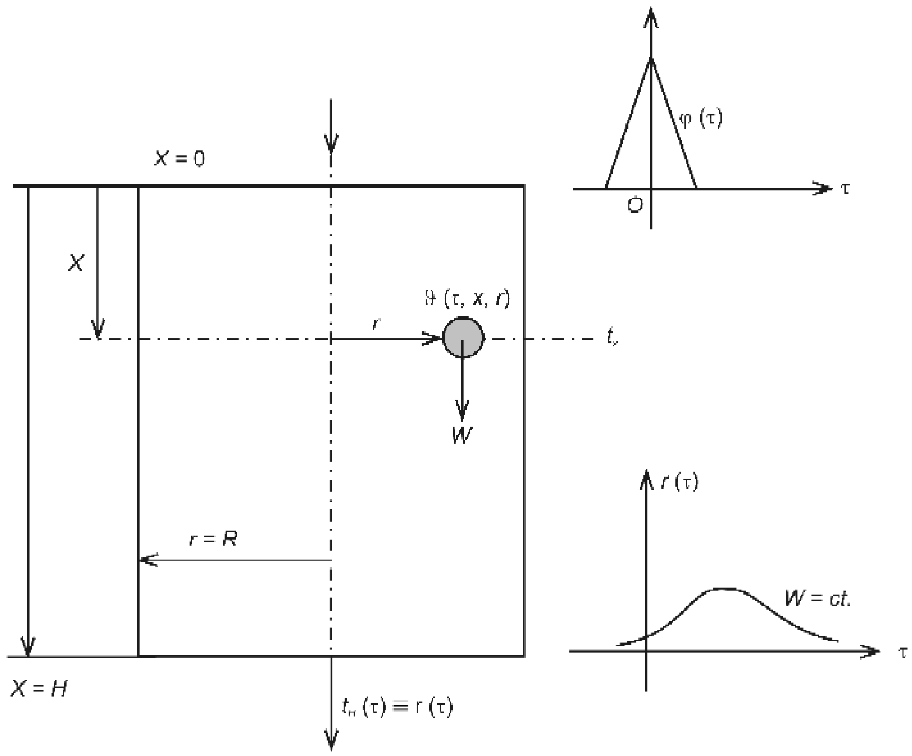

Figure 1. Calculus diagram

the lateral and bottom surfaces of the STSU. For simplification, the environment temperature value may be considered equal to the initial temperature value of the STSU water.

For the upper part of the STSU, the heating heat carrier is supposed to be uniformly distributed on $x = 0$ coordinate surface. The mathematical function describing its temperature variation in time is the STSU actual excitation function. In order to obtain the STSU unitary thermal response, an impulsional function $f(\tau)$ will be used. Triangular impulse functions are to be used [3].

The mathematical expression of the problem is therefore the following:

$$\frac{\partial \vartheta}{\partial \tau} + w \cdot \frac{\partial \vartheta}{\partial x} = a \cdot \frac{\partial^2 \vartheta}{\partial x^2} + a \cdot \frac{\partial^2 \vartheta}{\partial r^2} + a \cdot \frac{1}{r} \cdot \frac{\partial \vartheta}{\partial r} \quad (1)$$

The initial condition

$$\vartheta(0, x, r) = t_s \quad (2)$$

Boundary conditions:

$$-\lambda \cdot \left.\frac{\partial \vartheta}{\partial r}\right|_{r=R} = K(\vartheta - t_s)\big|_{r=R} \quad (3_1)$$

$$-\lambda \cdot \left.\frac{\partial \vartheta}{\partial x}\right|_{x=H} = K(\vartheta - t_s)\big|_{x=H} \quad (3_2)$$

$$-\lambda \cdot \left.\frac{\partial \vartheta}{\partial r}\right|_{r=0} = 0 \quad (3_3)$$

$$\vartheta(\tau, 0, r) = f(\tau) + t_s \quad (3_4)$$

where $f(\tau)$ is the impulsional excitation function.

At STSU outlet, the water temperature is the average temperature value in section $x = H$:

$$t_H = \frac{2}{R^2} \cdot \int_0^R r\vartheta(H, r, \tau)\, \mathrm{d}r \quad (4)$$

A new variable is used instated of function $\vartheta(\tau, x, r)$:

$$\Theta(\tau, x, r) = \vartheta(\tau, x, r) - t_s \quad (5)$$

Using Laplace transformation and considering the initial condition, equation (1) becomes:

$$w \cdot \frac{\partial \overline{\Theta}}{\partial x} + p\overline{\Theta} = a \cdot \frac{\partial^2 \overline{\Theta}}{\partial x^2} + a \cdot \frac{\partial^2 \overline{\Theta}}{\partial r^2} + a \cdot \frac{1}{r} \cdot \frac{\partial \overline{\Theta}}{\partial r} \quad (6)$$

where

$$\overline{\Theta}(p, x, r) = \mathrm{L}\{\Theta(\tau, x, r)\} \quad (7)$$

In order to integrate equation (6), the variable separation method is used and the following solution is suggested:

$$\overline{\Theta} = X(x) \cdot R(r) \quad (8)$$

By replacing (8) in equation (6) and by separating variables $X(x)$ and $R(r)$, two ordinary differential equations are obtained:

$$a \cdot \frac{\mathrm{d}^2 X}{\mathrm{d}x^2} - w \cdot \frac{\mathrm{d}X}{\mathrm{d}x} - (p + \lambda_j^2) \cdot X = 0 \quad (9_1)$$

$$a \cdot \frac{\mathrm{d}^2 R}{\mathrm{d}r^2} + a \cdot \frac{1}{r} \cdot \frac{\mathrm{d}R}{\mathrm{d}r} + \lambda_j^2 \cdot R = 0 \quad (9_2)$$

where λ_j^2 represents eigenvalues.

The boundary conditions (3_1), (3_4) become:

$$-\lambda \cdot \left.\frac{\partial \overline{\Theta}}{\partial r}\right|_{r=R} = K \cdot \overline{\Theta}\big|_{r=R} \quad (10_1)$$

$$-\lambda \cdot \left.\frac{\partial \overline{\Theta}}{\partial x}\right|_{x=H} = K \cdot \overline{\Theta}\big|_{x=H} \quad (10_2)$$

$$-\lambda \cdot \left.\frac{\partial \overline{\Theta}}{\partial r}\right|_{r=0} = 0 \quad (10_3)$$

$$\overline{\Theta}(p, 0, r) = \varphi(p) \quad (10_4)$$

Equation (9_2) is reduced to a Bessel equation whose solution is:

$$R\left(\frac{\lambda_j}{\sqrt{a}} \cdot r\right) = C_1 J_0\left(\frac{\lambda_j}{\sqrt{a}} \cdot r\right) + C_2 Y_0\left(\frac{\lambda_j}{\sqrt{a}} \cdot r\right) \quad (11)$$

By using boundary conditions (10_1) and (10_3), the eingenvalue equation is obtained:

$$J_0\left(\frac{\lambda_j}{\sqrt{a}}\cdot R\right)=J_1\left(\frac{\lambda_j}{\sqrt{a}}\cdot R\right)\cdot\frac{\lambda_j}{\sqrt{a}}\cdot R\cdot\frac{1}{Bi} \tag{12}$$

where *Bi* is Biot's dimensionless number determined for water soil overall heat transfer. The solutions of equation (12) are obtained for each particular case to be analysed. For example, considering STSU with 250 m^3 capacity and 2 m height, for several *Bi* values, the following first five eigenvalues results:

Bi	**δ [m]**	v_1	v_2	v_3	v_4	v_5
2.33	0.2	1.60	4.36	7.40	10.40	13.44
1.17	0.4	1.26	4.10	7.20	10.25	13.44
0.82	0.6	1.12	4.00	7.20	10.25	13.44
0.58	0.8	1.00	3.95	7.20	10.25	13.44
0.47	1.0	0.92	3.90	7.20	10.25	13.44
0.39	1.2	0.85	3.90	7.20	10.25	13.44
0.34	1.4	0.80	3.90	7.20	10.25	13.44

where:

$$v_j=\frac{\lambda_j}{\sqrt{a}}\cdot R$$

δ – STSU heat insulation thickness [m].

The solution of equation (9_1) is

$$X(x) = C_3\cdot\exp(m_1 x) + C_4\cdot\exp(m_2 x) \tag{13}$$

where m_1 and m_2 are the characteristic equation solutions:

$$m_1=\frac{1}{R}\cdot\left[\frac{1}{4}\cdot Pe+\sqrt{\left(\frac{1}{4}\cdot Pe\right)^2+\frac{R^2}{a}\cdot(p+\lambda_j^2)}\right] \tag{13_1}$$

$$m_2=\frac{1}{R}\cdot\left[\frac{1}{4}\cdot Pe-\sqrt{\left(\frac{1}{4}\cdot Pe\right)^2+\frac{R^2}{a}\cdot(p+\lambda_j^2)}\right] \tag{13_2}$$

where: *Pe* is Peclet's dimensionless number. The two solutions $X(x)$ and $R(r)$ are grouped under the form of solution (8) and

$$\overline{\Theta}(p,x,r)=\sum_j C_j\cdot\left\{\exp(m_1 x)-\frac{\lambda m_1+K}{\lambda m_2+K}\cdot\right.$$
$$\left.\cdot\exp[(m_1-m_2)\cdot H]\cdot\exp(m_2 x)\right\}\cdot J_0\left(\frac{\lambda_i}{\sqrt{a}}\cdot r\right) \tag{14}$$

is obtained, where:

C_j – eigenfunctions

Using boundary condition (10_4) and considering the orthogonalaty of Bessel's functions,

$$C_j=2\cdot\varphi(p)\cdot\frac{1}{v_j}\cdot J_1^{-1}(v_j) \tag{15}$$

is obtained.

The transformed solution of equation (1) will ultimately have the following expression:

$$\overline{\Theta}(p,\tilde{x},\tilde{r})=\sum_j\{\exp[C(\tilde{x})+D_1(\tilde{x})\cdot\sqrt{A+Bp}]-$$
$$\left.-\frac{E+\sqrt{A+Bp}}{E-\sqrt{A-Bp}}\cdot\exp[C(\tilde{x})-D_2(\tilde{x})\cdot\sqrt{A+Bp}]\right\}\cdot$$
$$\cdot\frac{\varphi(p)}{v_j}\cdot\frac{J_0(v_j\tilde{r})}{J_1(v_j)} \tag{16}$$

where

$$A=\left(\frac{1}{4}\cdot Pe\right)^2+v_j^2$$

$$B=\frac{R^2}{a}$$

$$C(\tilde{x})=\frac{1}{4}\cdot(\tilde{X}\tilde{H})\cdot Pe$$

$$D_1(\tilde{x})=\tilde{X}\tilde{H}$$

$$D_2(\tilde{x})=(\tilde{X}-2)\cdot\tilde{H}$$

$$E=\frac{1}{4}\cdot Pe+Bi$$

$$\tilde{x}=\frac{X}{H}$$

$$\tilde{H} = \frac{H}{R}$$

$$\tilde{r} = \frac{r}{R}$$

The original solution will be obtained either by using the theorem of residues or by using Laplace transformation conversion tables:

$$\Theta(\tau, \tilde{H}, \tilde{r}) = \sum_j \Theta_1(\tau, \tilde{H}, \tilde{r}) + \sum_j \Theta_2(\tau, \tilde{H}, \tilde{r}) + \sum_j \Theta_3(\tau, \tilde{H}, \tilde{r}) \tag{17}$$

where:

$$\Theta_1(\tau, \tilde{H}, \tilde{r}) = \exp C(\tilde{H}) \cdot \frac{1}{v_j} \cdot \frac{J_0(v_j \tilde{r})}{J_1(v_j)} \cdot \left\{ \varphi(\tau) * \left\{ \frac{\overline{D}_1(\tilde{H})}{2\sqrt{\pi\tau^3}} \cdot \exp\left\{ -\left[\overline{A} \cdot \tau + \frac{\overline{D}_1^{\,2}(\tilde{H})}{4\tau} \right] \right\} \right\} \right\} \tag{17$_1$}$$

$$\Theta_2(\tau, \tilde{H}, \tilde{r}) = -\exp C(\tilde{H}) \cdot \frac{1}{v_j} \cdot \frac{J_0(v_j \tilde{r})}{J_1(v_j)} \cdot \left\{ \varphi(\tau) * \left\{ \frac{\overline{D}_2(\tilde{H})}{2\sqrt{\pi\tau^3}} \cdot \exp\left\{ -\left[\overline{A} \cdot \tau + \frac{\overline{D}_2^{\,2}(\tilde{H})}{4\tau} \right] \right\} \right\} \right\} \tag{17$_2$}$$

$$\Theta_3(\tau, \tilde{H}, \tilde{r}) = 2E\sqrt{B} \cdot \exp C(\tilde{H}) \cdot \frac{1}{v_j} \cdot \frac{J_0(v_j \tilde{r})}{J_1(v_j)} \cdot \left\{ \varphi(\tau) * \left\{ \frac{F}{\sqrt{\pi\tau}} \cdot \exp\left\{ -\left[\overline{A} \cdot \tau + \frac{\overline{D}_2^{\,2}(\tilde{H})}{4\tau} \right] \right\} + F^2 \exp(L\tau + M) \cdot erfc\left(\frac{M}{2E\sqrt{\tau / B}} - E\sqrt{\frac{\tau}{B}} \right) \right\} \right\} \tag{17$_3$}$$

where

* – convolution product

$$\overline{D}_1(\tilde{H}) = -D_1(\tilde{H}) \cdot \sqrt{B}$$

$$\overline{A} = A / B$$

$$\overline{D}_2(\tilde{H}) = -D_2(\tilde{H}) \cdot \sqrt{B}$$

$$F = E / \sqrt{B}$$

$$L = F^2 - \overline{A}$$

$$M = F \cdot \overline{D}_2(\tilde{H})$$

The temperature pattern profile inside STSU at each moment τ is obtained using relation (17).

Considering relation (5), the STSU water outlet temperature is determined using (4).

We note:

$$r(\tau) = \frac{2}{R^2} \cdot \int_0^R r \cdot \vartheta(H, r, \tau)\, \mathrm{d}r \tag{18}$$

STSU thermal response.

If the actual excitation of STSU is represented by a time function $F(\tau)$, the STSU thermal response at this type of excitation is determined using relation:

$$R(\tau) = r(\tau) * F(\tau) \tag{19}$$

The excitation function is represented by the temperature variation of the heat carrier introduced in the STSU; the heat carrier is heated by the solar collectors.

III. STSU transient analysis

In order to determine the efficiency of a STSU, the temperature pattern at any moment should be know, mainly at the end of the heat accumulation period. Supposing that the solar installation is equipped with a solar radiation collecting area S_p, according to the heat carrier specific flow-rate, to the geometrical and physical characteristics of the STSU and to the radiative climate determined by the equivalent temperature $t_E(\tau)$, the heat quantity introduced in the STSU is calculated using relation

$$Q_s = \eta \cdot I \cdot S_p \tag{20}$$

The solar radiation collecting efficiency is determined using the solar collector characteristic equation, according to the heat carrier average temperature or to the heat carrier temperature when introduced in the solar collector.

$$\eta = F_R \cdot K_\Sigma \cdot [t_E(\tau) - t_R(\tau)] / I \qquad (21)$$

For short or well insulated pipes, the following approximation may be accepted:

$$t_R(\tau) = t_H(\tau)$$

The STSU thermal excitation function is $t_c(\tau)$. Considering (19) and the solar collector characteristic equation, the STSU actual thermal response results:

$$t_R(\tau) = \frac{r(0)}{1 - r(0) \cdot (1 - F_R \cdot K_\Sigma / \dot{m} \cdot c_p)} \cdot$$

$$\cdot \left[\frac{F_R \cdot K_\Sigma}{\dot{m} \cdot c_p} \cdot t_E(\tau) + \sum_{j=1}^{j-1} r(j) \cdot t_c(\tau - j) \right] \qquad (22)$$

where $r(j)$ – STSU unit thermal response.

The heat quantity accumulated during the service period is determined using relation

$$Q_{AC} = \pi \cdot R^2 \cdot H \cdot \rho \cdot g \cdot c_p \cdot (\bar{t} - t_s) \qquad (23)$$

where:

$\bar{t}$ – average temperature value of the storage tank water when the heat accumulation period is over.

STSU efficiency is calculated using relation

$$\varepsilon = \frac{Q_{AC}}{Q_S} \qquad (24)$$

Function ε is defined according to the independent values conditioning the STSU thermal performance. Decisions on the technical and economic optimization of STSU solar heating system may therefore be taken.

IV. Conclusions

STSU thermal response and accordingly season service efficiency may be determined using the method described above. Solar installation dimensioning represents the result of long-period dynamic analyses. The mathematical model, the analytical solution and the practical method are valid for the analysis of large central solar installations.

In case of small installations, such an analysis is complicated and not efficient considering the computer operational time.

The solution introduced in this paper is part of an INCERC calculus program, to be used in analyzing urban planning solutions for towns with seasonal warm water producing solar installations (service period: May-October).

Nomenclature

t_{ext} – environment temperature [°C]
t_s – storage tank water initial temperature [°C]
$t_E(\tau)$ – sol-air temperature [°C]
$t_R(\tau)$ – water temperature at solar collector inlet [°C]
$t_c(\tau)$ – water temperature at solar collector outlet [°C]
ϑ – temperature function
Q_{AC} – heat quantity supplied and stored [kWh]
Q_s – heat quantity admitted in storage tank [kWh]
W – storage tank water flow velocity [m / s]
K – water – environment heat loss overall coefficient [W / m²K]
K_Σ – solar collector heat loss overall coefficient [W / m²K]
I – solar radiation intensity [W / m²]
$\dot{m}$ – water specific flow rate in solar collectors [kg/m²s]
a – water heat diffusivity [m² / s]
λ – water thermal conductibility [W / mK]
c_p – water specific heat [J / kgK]
ρ – water density [kg / m³]
R – storage tank radius [m]
H – storage tank height [m]
S_p – solar radiation collecting area [m²]
F_R – solar collector heat removal factor
λ_j – eigenvalues
η – solar radiation collecting efficiency
p – complex variable
Bi – Biot's dimensionless number
Pe – Peclet's dimensionless number

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